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Publisher's Editorial

A New Memory

Solomon A. Garfunkel
Executive Director
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This past year, COMAP began publication of a new journal. While unlike almost all other COMAP products and publications, this journal did not originate with a specific COMAP project—rather, it came to us from the editors. We are excited about the innovative and unique nature of this publication. We are proud to unabashedly promote it here.

The *International Journal for the History of Mathematics Education* is the only journal that is entirely devoted to the world history of mathematics education. The major aim of this journal is to provide mathematics education with its *memory*, in order to reveal the insights achieved in earlier periods (ranging from ancient times to the late 20th century) and to unravel the fallacies of past events.

The journal is published twice a year and each issue is roughly 100 pages long. It features research papers, notes, book reviews, and interviews. The Chief Editor is Gert Schubring, Bielefeld University, Germany; the Managing Editor is Alexander Karp, Teachers College, Columbia University, USA.

Editorial Board

The UMAP Journal 29 (2) (2008) 93–95.

Excerpt

Here is an excerpt from our third issue, December 2007, in which we interviewed Dr. Henry Pollak (former Director of Mathematics and Statistics Research at Bell Laboratories, now at Teachers College, Columbia University), and he recounted some of his experiences through so many years in the field.

Henry Pollak.

> There was the famous 1963 letter [Bers et al. 1962] that complained about the "New Math," it was signed and published in both the *American Mathematical Monthly* and by NCTM [National Council of Teachers of Mathematics]. I signed that letter, as did one other person from SMSG [School Mathematics Study Group], and I did it for a very good reason, that is that I thought SMSG satisfied the conditions that had been written down. [Morris] Kline and [Max M.] Schiffer and others didn't think so, but I did and it's not that I was so crazy. It's that I lived in the middle of discrete mathematics. I lived at the forefront of understanding how important that is for applications. So the structural part of SMSG was beautiful preparation for discrete mathematics. And the trouble is that people forget that telephony involves not just transmission, which is classical mathematical physics, but also switching, which was discrete mathematics in its most difficult form. Terribly difficult and terribly interesting. And so I lived in both of these, Bell Labs and SMSG, you see, and so to me, what was going on in SMSG was beautiful for the modern applications of mathematics, and I think that the classical applied mathematicians who signed this letter didn't understand this. It's extremely difficult to get over the idea that something could be as important as the thousands of years of success of applying mathematics to physics. I mean, classical analysis is an enormous edifice of success, and yet the success of discrete mathematics in the last fifty years is enormous as well. So I think that

> that is why I signed that letter. And nobody in SMSG ever complained about my signing that letter [laughs], because they understood. But I think a lot of the people at the time didn't understand how applied mathematics was changing. Nowadays, and even by 1963 the definition of applied mathematics is no longer just classical analysis applied to classical physics. It is a lot more than that. And when you look at all of applicable mathematics SMSG is excellent preparation for that. It's exactly what you want. [Karp 2007, 74–75]

More detailed information about the journal can be found at

`http://www.comap.com/historyjournal/index.html`

References

Bers, Lipman, et al. 1962. On the mathematics curriculum of the high school. *American Mathematical Monthly* 69 (3) (March 1962): 189–193.

Karp, Alexander. 2007. Interview with Henry Pollak. *The International Journal for the History of Mathematics Education* 2 (2): 67–89. `http://www.comap.com/historyjournal/archives.htm`.

About the Author

Solomon Garfunkel, previously of Cornell University and the University of Connecticut at Storrs, has dedicated the last 25 years to research and development efforts in mathematics education. He has served as project director for the Undergraduate Mathematics and Its Applications (UMAP) and the High School Mathematics and Its Applications (HiMAP) Projects funded by NSF, and directed three telecourse projects including *Against All Odds: Inside Statistics*, and *In Simplest Terms: College Algebra*, for the Annenberg/CPB Project. He has been the Executive Director of COMAP, Inc. since its inception in 1980. Dr. Garfunkel was the project director and host for the series, *For All Practical Purposes: Introduction to Contemporary Mathematics*. He was the Co-Principal Investigator on the ARISE Project, and is currently the Co-Principal Investigator of the CourseMap, ResourceMap, and WorkMap projects. In 2003, Dr. Garfunkel was Chair of the National Academy of Sciences and Mathematical Sciences Education Board Committee on the Preparation of High School Teachers.

Modeling Forum

Results of the 2008 Interdisciplinary Contest in Modeling

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Division Chief, Mathematical Sciences Division
Program Manager, Cooperative Systems
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Introduction

A record total of 380 teams from three countries spent a weekend in February working in the 10th Interdisciplinary Contest in Modeling (ICM). They confronted an open-ended interdisciplinary modeling problem involving public health policy concerning healthcare systems. This year's contest began on Thursday, Feb. 14 and ended on Monday, Feb. 18, 2008. During that time, teams of up to three undergraduate or high school students researched, modeled, analyzed, solved, wrote, and submitted their solutions. After the weekend of challenging and productive work, the solution papers were sent to COMAP for judging. Three of the top papers, which were judged to be Outstanding by the expert panel of judges, appear in this issue of *The UMAP Journal*.

COMAP's Interdisciplinary Contest in Modeling (ICM) along with it sibling contest, the Mathematical Contest in Modeling (MCM), involves students working in teams to model and analyze an open problem. Centering its educational philosophy on mathematical modeling, COMAP supports the use of mathematical tools to explore real-world problems. It serves society by developing students as problem-solvers in order to become better informed and prepared as citizens, contributors, consumers, workers, and community leaders. The ICM and MCM are examples of COMAP's efforts in working towards its goals.

The UMAP Journal 29 (2) (2008) 97–111.

This year's public health problem was challenging in its demand for teams to utilize many aspects of science, mathematics, and data analysis in their modeling. The problem required teams to understand the data and nature of healthcare systems and to model the complex policy issues associated with the issues of financing and managing a nation's healthcare system. In order to accomplish their tasks, the students had to consider many difficult and complex issues. Political, social, psychological, and technological issues had to be considered, along with several challenging requirements needing scientific and mathematical analysis. The problem also included the ever-present requirements of the ICM to use thorough data analysis, research, creativity, and effective communication. The author of the problem was Kathleen Crowley, Professor of Psychology at the College of Saint Rose.

All members of the 380 competing teams are to be congratulated for their excellent work and dedication to modeling and problem solving. The judges remarked that this year's problem was challenging and demanding in many aspects of modeling and problem solving.

Next year, we will move to an environmental science theme for the contest problem. Teams preparing for the 2009 contest should consider reviewing interdisciplinary topics in the area of environmental issues.

The Healthcare Problem

Finding the Good in Healthcare Systems

Nations have systems for providing healthcare for their residents. Issues that are often of concern to people and are often in the news include which system is better and whether current systems can be improved. Aspects of these systems vary widely between nations: how they are funded; whether services are delivered through public, private, or non-profit organizations; whether public insurance is universal for all residents; who is eligible for assistance; what care is covered; whether the latest medical procedures are available; and how much is required as user fees. Other factors that are often debated in determining the quality of care include: coverage for complementary care (glasses, dental, prostheses, prescription drugs, etc.); which diseases are the most critical in improving overall health; percentage of GDP spent on healthcare; percentage of healthcare costs that goes toward labor/administration/malpractice insurance; ratio of public to private spending on healthcare; per capita spending on healthcare; growth of per capita spending on healthcare; number of participating physicians; per capita sick days; fairness of care in terms of age, race, gender, socio-economic class; and many more. Adding to the complications are health related factors such as personal exercise, food availability, climate, occupations of citizens, and smoking habits.

The World Health Organization (WHO), an agency of the United Nations, is a source of data on health factors. The annual World Health Report `http://www.who.int/whr/en/index.html`

assesses global health factors, and World Health Statistics

`http://en.wikipedia.org/wiki/World_Health_Organisation`

provides health statistics for the countries in the U.N. The production and dissemination of health statistics is a major function of WHO. To many people, these data and the associated analyses are considered unbiased and very valuable to the world community. There are many other sources of reliable health data available.

Part I: Describe several different outcomes (metrics) that could be used to evaluate the effectiveness of a country's healthcare system, such as average life expectancy of its residents. What metric would you use to make comparisons between existing and potential systems? Can you combine your metrics to make them even more useful in measuring quality?

Part II: Identify current sources of data that provide the raw data needed to compute the metrics you have identified above. You may need to modify your list of metrics based on the availability of data. Explain why you have selected those data and demonstrate how they can be used to assess and compare the relative effectiveness of healthcare systems as they exist in different countries.

Part III: Choose at least three of the most important and viable metrics for comparing healthcare systems. Justify why these are the most useful for this purpose. Can any of these help measure the historical change in an existing healthcare system? Are they measurable and can the data be easily collected?

Part IV: Use your three (or more) metrics to compare the United States healthcare system with one other country which is considered to have good healthcare using the most recent year for which you have data. Which country has the better healthcare system? Is your answer definitive?

Part V: Using your metrics compare the United States and one other country which is considered to have poor healthcare using the most recent year for which you have data. Which country has the better healthcare system?

Part VI: Pick a country's (U.S. or other) healthcare system and restructure it to improve the system based on your metrics. Build predictive models to test various changes to determine if the changes will improve the overall quality of the system. Suggest major change(s) that can improve the system.

The Results

The 380 solution papers were coded at COMAP headquarters so that names and affiliations of the authors were unknown to the judges. Each paper was then read preliminarily by "triage" judges at the U.S. Military Academy at West Point, NY. At the triage stage, the summary, the model description, and overall organization are the primary elements in judging a paper. Final judging by a team of modelers, analysts, and subject-matter experts took place in April. The judges classified the 380 submitted papers as follows:

	Outstanding	Meritorious	Honorable Mention	Successful Participation	Total
Healthcare	3	53	175	149	380

The three papers that the judges designated as Outstanding appear in this special issue of *The UMAP Journal*, together with a commentary by the judges. We list those three Outstanding teams and the 53 Meritorious teams (and advisors) below. The complete list of all participating schools, advisors, and results is provided in the **Appendix**.

Outstanding Teams

Institution and Advisor	**Team Members**
"Evaluation and Improvement of Healthcare Systems"	
Beijing University of Posts and Telecommunications	Luting Kong
Beijing, China	Yiyi Chen
Qing Zhou	Chao Ye
"The Most Expensive Is Not the Best"	
National University of Defense Technology	Hongxing Hao
Changsha, China	Xiangrong Zeng
Ziyang Mao	Boliang Sun
"Better Living through Math: An Analysis of Healthcare Systems"	
Harvey Mudd College	Denis Aleshin
Claremont, CA	Bryce Lampe
Darryl Yong	Parousia Rockstroh

Meritorious Teams (53)

Anhui University, China (Huayou Chen)
Asbury College, Wilmore, KY (Duk Lee)
Beijing Jiaotong University, China (Hong Zhang)
Beijing University of Posts, China (Tianping Shuai)
Beijing Language and Culture University, China (Xiaoxia Zhao)
Central University of Finance and Economics, China (Zhaoxu Sun)
China University of Mining and Technology, China (2 teams) (Xingyong Zhang) (Shujuan Jiang)
Chongqing University, China (2 teams) (Renbin He) (Jian Xiao)
College of Science of Harbin Engineering, China (Yu Fei)
Dalian University of Technology, China (4 teams) (Mingfeng He) (Liang Zhang) (Zhe Li) (Shengjun Xu)
Donghua University, China (Xiaofeng Wang)
East China University of Science and Technology, China (Lu Xiwen)

Fudan University, School of Management, China (Zhongyi Zhu)
Hangzhou Dianzi University, China (2 teams) (Zhifeng Zhang) (Chengjia Li)
Harbin Institute of Technology, China (3 teams) (Zhu Lei) (Xiaofeng Shi)
(Xuefeng Wang)
Harbin University of Science and Technology, China (Fengqiu Liu)
Harvey Mudd College, Claremont, CA (Darryl Yong)
Huazhong University of Science and Technology, China (Yan Dong)
James Madison University, Harrisonburg, VA (Hasan Hamdan)
Jinan University, China (Shizhuang Luo)
Nanjing University, China (2 teams) (Huikun Jiang) (Li Wei Xu)
National University of Defense Technology, China (2 teams) (Mengda Wu) (Yi Wu)
Peking University, China (2 teams) (Yuxin Liu) (Yuan Wang)
Peking University Health Sciences Center, China (Termison)
Sichuan University, China (Hai Niu)
Sichuan Agricultural University, China (Shi Du)
South China Normal University, China (Xiuxiang Liu)
South China Agricultural University, China (YanKe Zhu)
South China University of Technology, China (2 teams) (WeiJian Ding)
(ShenQuan Liu)
Shandong University at Weihai, China (Zhulou Cao)
Tsinghua University, China (Mei Lu)
University of Science and Technology of China, China (2 teams) (Hong Zhang)
(Yuanbo Zhang)
Xi'an Jiaotong University, China (Zhuosheng Zhang)
Yuanpei College, China (Liman Sha)
Zhejiang University, China (2 teams) (Qifan Yang) (Zhiyi Tan)
Zhejiang Gongshang University, China (Zhu Ling)
Zhuhai College of Jinan University China (3 teams) (YuanBiao Zhang)
(Zhiwei Wang) (Yuanbiu Zhang)

Awards and Contributions

Each participating ICM advisor and team member received a certificate signed by the Contest Directors and the Head Judge. Additional awards were presented to the National University of Defense Technology team from the Institute for Operations Research and the Management Sciences (INFORMS).

Judging

Contest Directors
Chris Arney, Division Chief, Mathematical Sciences Division,
Army Research Office, Research Triangle Park, NC
Joseph Myers, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY

Associate Director
Rodney Sturdivant, Dept. of Mathematical Sciences,
U.S. Military Academy, West Point, NY

Judges
Ben Cole, National Security Agency, Ft. Meade, MD
William C. Dowdy, U.S. Army Medical Department Activity,
West Point, NY
John Kobza, Dept. of Industrial Engineering, Texas Tech University,
Lubbock, TX
Sarah Root, Dept. of Industrial Engineering, University of Arkansas,
Fayetteville, AR
Frank Wattenberg, Dept. of Mathematical Sciences, U.S. Military Academy,
West Point, NY

Triage Judges
Dept. of Mathematical Sciences, U.S. Military Academy, West Point, NY:

Amanda Beecher, Randy Boucher, Robert Burks, Gabriel Costa, Jong Chung, Ben Cole, Brian Davis, Eric Drake, J. Dzwonchyk, Amy Erickson, Keith Erickson, Douglas Fletcher, Gregory Graves, Michael Harding, Alex Heidenberg, Heather Jackson, Anthony Johnson, Jerry Kobylski, Elizabeth Morseman, Joseph Myers, Donald Outing, Jack Picciuto, Todd Retchless, Jon Roginski, Tyler Smith, Brian Souhan, Rodney Sturdivant, Patrick Sullivan, Edward Swim, Krista Watts, Brian Winkel, and Robyn Wood

Acknowledgments

We thank:

- INFORMS, the Institute for Operations Research and the Management Sciences for its support in judging and providing prizes for the INFORMS winning team;
- IBM for their support for the contest;
- all the ICM judges and ICM Board members for their valuable and unflagging efforts;
- the staff of the U.S. Military Academy, West Point, NY, for hosting the triage and final judgings.

Cautions

To the reader of research journals:

Usually a published paper has been presented to an audience, shown to colleagues, rewritten, checked by referees, revised, and edited by a journal editor. Each of the student papers here is the result of undergraduates working on a problem over a weekend; allowing substantial revision by the authors could give a false impression of accomplishment. So these papers are essentially au naturel. Light editing has taken place: minor errors have been corrected, wording has been altered for clarity or economy, style has been adjusted to that of *The UMAP Journal*, and the papers have been edited for length. Please peruse these student efforts in that context.

To the potential ICM Advisor:

It might be overpowering to encounter such output from a weekend of work by a small team of undergraduates, but these solution papers are highly atypical. A team that prepares and participates will have an enriching learning experience, independent of what any other team does.

Editor's Note

As usual, the Outstanding papers were longer than we can accommodate in the *Journal*, so space considerations forced me to edit them for length. It was not possible to include all of the many tables and figures.

In editing, I endeavored to preserve the substance and style of the paper, especially the approach to the modeling.

—Paul J. Campbell, Editor

Appendix: Successful Participants

KEY:
P = Successful Participation
H = Honorable Mention
M = Meritorious
O = Outstanding (published in this special issue)

INSTITUTION	DEPT.	CITY	ADVISOR	C
CALIFORNIA				
Cal. State U. at Monterey Bay	Science and Env'l Policy	Seaside	Herbert Cortez	P
Harvey Mudd College	Mathematics	Claremont	Darryl Yong	O
Harvey Mudd College	Mathematics	Claremont	Darryl Yong	M
Harvey Mudd College	Mathematics	Claremont	Rachel Levy	H
Univ. of California, Davis	Mathematics	Davis	Karl Beutner	P
IOWA				
Simpson College	Mathematics	Indianola	William Schellhorn	H
Simpson College	Mathematics	Indianola	Debra Czarneski	P
Simpson College	Mathematics	Indianola	Rick Spellerberg	P
KENTUCKY				
Asbury College	Mathematics and CS	Wilmore	Duk Lee	H
Asbury College	Mathematics and CS	Wilmore	Duk Lee	M
Asbury College	Mathematics and CS	Wilmore	Ken Rietz	H
MARYLAND				
Villa Julie College	Mathematics	Stevenson	Eileen McGraw	P
MONTANA				
Carroll College	Math/Eng'ng/CS	Helena	Kelly Cline	H
Carroll College	Math/Eng'ng/CS	Helena	Kelly Cline	P
Carroll College, MT	Math/Eng'ng/CS	Helena	Mark Parker	P
NEW JERSEY				
Princeton University	Mathematics	Princeton	Ingrid Daubechies	H
NEW YORK				
Clarkstown HS South	Mathematics	West Nyack	Mary Gavioli	P
NORTH CAROLINA				
Duke University	Mathematics	Durham	Bianca Santoro	P
VIRGINIA				
James Madison University	Math and Stat	Harrisonburg	Hasan Hamdan	M
Virginia Commonwealth U.	Biomedical Engineering	Richmond	Dianne Pawluk	H
WASHINGTON				
Seattle Pacific University	Electrical Engineering	Seattle	Melani Plett	P
HONG KONG				
City Univ. of Hong Kong	Management Sciences	Hong Kong	Ligang Zhou	P
Hong Kong Baptist Univ.	Mathematics	Kowloon	Man Lai Tang	P
INDONESIA				
Institut Teknologi Bandung	Mathematics	Bandung	Edy Soewono	P
Institut Teknologi Bandung	Mathematics	Bandung	Kuntjoro Sidarto	H

INSTITUTION	DEPT.	CITY	ADVISOR	C
CHINA				
Anhui				
Anhui University	Statistics	Hefei	Huayou Chen	M
Anhui University	Statistics	Hefei	Ligang Zhou	H
Hefei University of Technology	Applied Mathematiccs	Hefei	Yongwu Zhou	H
Hefei University of Technology	Comp'l Mathematics	Hefei	Youdu Huang	P
University of Sci. & Tech. of China	Gifted Young	Hefei	Yuanbo Zhang	M
University of Sci. & Tech. of China	Statistics and Finance	Hefei	Hong Zhang	M
Beijing				
Beihang University	Electronic Engineering	Beijing	Wu Sanxing	H
Beijing Electron. Sci. & Tech. Inst.	Basic Education	Beijing	Peiqun Wu	H
Beijing Institute of Technology	Mathematics	Beijing	Hua-Fei Sun	H
Beijing Institute of Technology	Mathematics	Beijing	Bing-Zhao Li	P
Beijing Institute of Technology	Mathematics	Beijing	Guifeng Yan	H
Beijing Institute of Technology	Mathematics	Beijing	Chunguang Xiong	H
Beijing Institute of Technology	Mathematics	Beijing	Qun Ren	P
Beijing Institute of Technology	Mathematics	Beijing	Xiuling Ma	P
Beijing Jiaotong University	Applied Mathematics	Beijing	Jing Zhang	H
Beijing Jiaotong University	Chemistry	Beijing	Yongsheng Wei	H
Beijing Jiaotong University	Chemistry	Beijing	Yongsheng Wei	P
Beijing Jiaotong University	Mathematics	Beijing	Bingtuan Wang	H
Beijing Jiaotong University	Mathematics	Beijing	Zhouhong Wang	H
Beijing Jiaotong University	Mathematics	Beijing	Zhouhong Wang	H
Beijing Jiaotong University	Mathematics	Beijing	Shangli Zhang	P
Beijing Jiaotong University	Mathematics	Beijing	Jun Wang	H
Beijing Jiaotong University	Mathematics	Beijing	Xiaoming Huang	H
Beijing Jiaotong University	Mathematics	Beijing	Hong Zhang	M
Beijing Jiaotong University	Mathematics	Beijing	Pengjian Shang	H
Beijing Jiaotong University	Mathematics	Beijing	Keqian Dong	P
Beijing Jiaotong University	Mathematics	Beijing	Faen Wu	P
Beijing Jiaotong University	Mathematics	Beijing	Xiaoxia Wang	P
Beijing Jiaotong University	Mathematics	Beijing	Zhonghao Jiang	H
Beijing Jiaotong University	Mathematics	Beijing	Liang Wang	P
Beijing Jiaotong University	Mathematics	Beijing	Guozhong Liu	P
Beijing Jiaotong University	Mathematics	Beijing	Guozhong Liu	H
Beijing Jiaotong University	Mathematics	Beijing	Xun Chen	P
Beijing Jiaotong University	Mathematics	Beijing	Xun Chen	P
Beijing Jiaotong University	Physics	Beijing	Bingtuan Wang	P
Beijing Jiaotong University	Physics	Beijing	Bingtuan Wang	H
Beijing Jiaotong University	Statistics	Beijing	Weidong Li	H
Beijing Language and Culture U.	Computer Science	Beijing	Guilong Liu	H
Beijing Language and Culture U.	Computer Science	Beijing	Yanbing Feng	H
Beijing Language and Culture U.	Computer Science	Beijing	Yanbing Feng	H
Beijing Language and Culture U.	Information Sciences	Beijing	Xiaoxia Zhao	M
Beijing Language and Culture U.	Information Sciences	Beijing	Xiaoxia Zhao	P
Beijing Language and Culture U.	Information Sciences	Beijing	Ping Yang	H
Beijing Normal University	Mathematical Sciences	Beijing	Zhengru Zhang	H
Beijing U. of Posts & Telecomm.	Applied Mathematics	Beijing	Zuguo He	H
Beijing U. of Posts & Telecomm.	Applied Physics	Beijing	Jinkou Ding	P
Beijing U. of Posts & Telecomm.	Applied Physics	Beijing	Jinkou Ding	P
Beijing U. of Posts & Telecomm.	Communication Eng'ng	Beijing	Zuguo He	H
Beijing U. of Posts & Telecomm.	CS and Technology	Beijing	Hongxiang Sun	P
Beijing U. of Posts & Telecomm.	Economics and Mgmt	Beijing	Tianping Shuai	M
Beijing U. of Posts & Telecomm.	Electronic Engineering	Beijing	Qing Zhou	O
Beijing U. of Posts & Telecomm.	Electronics Info. Eng'ng	Beijing	Xinchao Zhao	H
Beijing U. of Posts & Telecomm.	Information Engineering	Beijing	Wenbo Zhang	H

INSTITUTION	DEPT.	CITY	ADVISOR	C
Beijing U. of Posts & Telecomm.	Information Engineering	Beijing	Wenbo Zhang	P
Beijing Univ. of Astro. & Aero.	Computer Science	Beijing	Hongying Liu	H
Beijing University of Chem. Tech.	Chemistry Science	Beijing	Kaisu Wu	P
Beijing University of Chem. Tech.	Mathematics	Beijing	Damin Liu	H
Beijing University of Chem. Tech.	Mathematics	Beijing	Wenyan Yuan	H
Central Univ. of Finance and Econ.	Applied Mathematics	Beijing	Xiaoming Fan	P
Central Univ. of Finance and Econ.	Applied Mathematics	Beijing	Zhaoxu Sun	M
Central Univ. of Finance and Econ.	Applied Mathematics	Beijing	Xianjun Yin	H
Central Univ. of Finance and Econ.	Applied Mathematics	Beijing	Donghong Li	H
China University of Geosciences	Information Technology	Beijing	Zhaodou Chen	H
China University of Geosciences	Information Technology	Beijing	Zhaodou Chen	H
China University of Geosciences	Mathematics	Beijing	Guangdong Huang	P
China University of Geosciences	Mathematics	Beijing	Yan Deng	P
China University of Geosciences	Mathematics	Beijing	Yan Deng	P
China University of Geosciences	Mathematics	Beijing	Linlin Zhao	P
Peking U., Health Sciences Ctr	Mathematics	Beijing	PKU Biomaths	H
Peking U., Health Sciences Ctr	Mathematics	Beijing	PKU Biomaths	M
Peking U., Health Sciences Ctr	Mathematics	Beijing	PKU Statistics	H
Peking U., Health Sciences Ctr	Mathematics	Beijing	PKU Statistics	H
Peking U., Health Sciences Ctr	Mathematics	Beijing	PKU Biostatistics	H
Peking U., Health Sciences Ctr	Mathematics	Beijing	PKU Biostatistics	H
Peking U., Health Sciences Ctr	Mathematics	Beijing	PKU Sciences	H
Peking U., Health Sciences Ctr	Mathematics	Beijing	PKU Sciences	H
Peking U., Health Sciences Ctr	Mathematics	Beijing	PKU Biosciences	H
Peking U., Health Sciences Ctr	Mathematics	Beijing	PKU Maths	P
Peking University	Extracurricular Activities	Beijing	Mingming Yu	H
Peking University	Microelectronics	Beijing	Yuan Wang	M
Peking University	Physics	Beijing	Yuxin Liu	M
Peking University	Probability and Statistics	Beijing	Minghua Deng	H
Peking University	Sci. and Eng'ng Computing	Beijing	Peng He	H
Peking University	Yuanpei College	Beijing	Liman Sha	M
Tsinghua University	Mathematical Sciences	Beijing	Mei Lu	M
Tsinghua University	Mathematical Sciences	Beijing	Mei Lu	P
Tsinghua University	Mathematical Sciences	Beijing	Jinxing Xie	P
U. of Int'l Business and Econ.	Int'l Trade and Econ.	Beijing	Baomin Dong	H
U. of Int'l Business and Econ.	Int'l Trade and Econ.	Beijing	Baomin Dong	P
Chongqing				
Chongqing University	Applied Chemistry	Chongqing	Zhiliang Li	P
Chongqing University	Applied Mathematics	Chongqing	Renbin He	M
Chongqing University	Infor. and Comp'l Science	Chongqing	Qu Gong	P
Chongqing University	Information and Comp'l Sci.	Chongqing	Jian Xiao	M
Chongqing University	Software Engineering	Chongqing	Bin Cai	P
Southwest University	Statistics	Chongqing	Xuegao Zheng	P
Southwest University	Statistics	Chongqing	Jianjun Yuan	H
Third Military Medical Univ.	Basic Medicine	Chongqing	Wanchun Luo	H
Third Military Medical Univ.	Mathematics	Chongqing	Mingkui Luo	P
Third Military Medical Univ.	Mathematics	Chongqing	Mingkui Luo	P
Fujian				
Fujian Agricult. and Forestry U.	Life Sciences	Fuzhou	Wu Long	P
Fujian Normal University	Computer Science	Fuzhou	Shenggui Zhang	P
Xiamen University	Mathematics	Xiamen	Li Shiyin	H
Guangdong				
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Evaluation and Improvement of Healthcare Systems

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Summary

To evaluate the effectiveness of healthcare systems, we describe metrics in three categories: resources, performance, and inequity. In the Incomplete-Induction Model, we use the Variance Analysis method to evaluate the significance of each metric. The four most important metrics are the percentage of GDP spent on healthcare, the ratio of general government expenditure on health to private expenditure, health-adjusted life expectancy, and health inequity.

We combine the metrics into two integrative metrics, the ratio of resources to performance, and health inequity, using the Analytical Hierarchy Process. The two metrics make up the Evaluation Vector.

To compare the effectiveness of different health systems by means of the Evaluation Vector, we construct two comparison models. In Model 1, we compare based on relative disparity. In Model 2, we introduce a coordinate system in which a vector stands for a healthcare system. The effectiveness of the system is reflected by the length of the vector: A smaller length stands for a better system.

In Task IV and Task V, we choose Brazil for its good healthcare system and India for its poor one. According to the two comparison models, both systems are better than that of the U.S. Then we analyze the relationship between resources and system effectiveness in order to explain why the Indian system is better.

In Task VI, we analyze the U.S. system and put forward suggestions

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to improve it. Then we build a model to investigate the influence of the changes. In addition, we measure the historical change in the system. Generally, its effectiveness is increasing, but the growth rate is slower recently.

We also analyze the strengths and weaknesses of each model.

Solution of Task 1

Description and Analysis

We put forward a method to measure a country's healthcare system. To simplify the problem, we first abstract the system as a simplified input-output system (**Figure 1**).

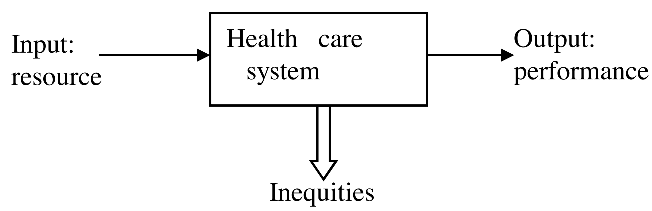

Figure 1. Healthcare as a simplified input-output system.

Sufficient resources should be put in to guarantee that the system functions well. Viewed in isolation, the more resources the system gets, the better it will be. However, linked to output, the better system is not the one with more resources but the one with a low input-output ratio. Later we discuss how to use the metric of resources to measure a healthcare system.

Output reflects the system's performance: The better the system is, the more output it will produce; we define performance later.

How the system operates can't be ignored, since that affects the whole health situation of the country, such as the distribution of resources and the health level in different areas. These factors will be expressed by the metric of Inequities.

Metrics

Resources

A good healthcare system needs adequate resources: human resources, material resources, and financial resources:

- **Human resources** are the population engaged in medical careers, including physicians, nurses, pharmacists, and other health workers.
- **Material resources** are the hardware facilities in the medical system, such as hospitals and hospital beds.

- **Financial resources** include three aspects:
  - The percentage of GDP spent on healthcare.
  - The percentage of total government expenditure spent on healthcare.
  - The ratio of government spending to private spending on health. Apparently, in a good health system this ratio is high.

Performance

- **Health level.** The main objective of a health system is improving health [WHO 2001]. We choose *disability-adjusted life expectancy (DALE)* and infant mortality as criteria, the combination of which can be used to evaluate the level of health.
  - Disability-adjusted life expectancy. DALE is the life expectancy at birth adjusted for disability [WHO 2001]. It is a comprehensive measure of the global burden of disease and the trends of population health level [Mathers et al. 2001].
  - Infant mortality rates. Infant mortality rate is a significant indicator of medical level: High-medical-level countries have a low infant mortality rate.
- **Health-service coverage.** Health-service coverage comprises several factors, such as the immunization coverage of 1-year-olds and the percentage of the population with public insurance. A good health system should provide healthcare for all of its citizens. Usually, developed countries have high rates in the both of those.
- **Responsiveness.** Responsiveness measures how the system performs relative to non-health aspects, meeting or not meeting a population's expectation of how it should be treated [WHO 2001]. The notion of responsiveness is composed of seven elements, including [WHO 2001]:
  - respect for dignity,
  - confidentiality,
  - autonomy to participate in choices about one's own health,
  - prompt attention,
  - amenities of adequate quality,
  - access to social support networks, and
  - freedom to select which individual or organization delivers one's care.

  The seven points above lead to a general metric of responsiveness. In part II we discuss how to combine them.

Inequities

- **Inequities in health.** A healthcare system is not so perfect if the health level varies widely between different categories of the population, even in countries with a rather good health status on average [WHO 2001].To describe inequities in health, we use life expectancy in terms of age, race, gender, socioeconomic class, and so on. If every category has the same life expectancy, the system is fair in terms of health level.

- **Inequities in responsiveness.** The same as health level: If some people are treated with courtesy and others are not, there are inequities in responsiveness.

- **Fairness of financial contribution.** To be fair, the expenditure each household faces should be distributed according to ability to pay rather than by risk of illness [WHO 2001]. That means that a household should not become impoverished to obtain healthcare, and rich households should pay more towards the system than poor households [Gakidou et al. 2000].

The Combination of Metrics

We devise a composite measure of the three metrics: Resources, Performance, and Inequities.

Analytical Hierarchy Process

- **Divide layers.** We divide the metrics into several layers as **Figures 2–5** show.

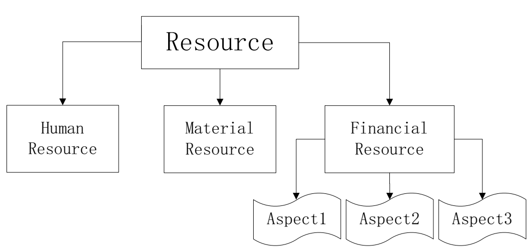

Figure 2. Resources.

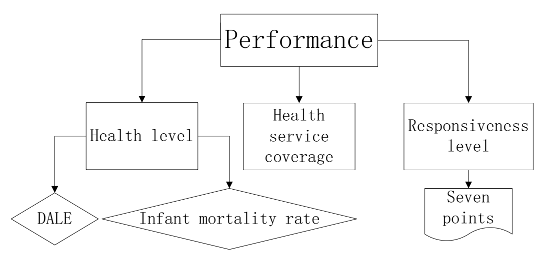

Figure 3. Performance.

- **Evaluation Vector.** A good system should use the least resources possible to produce performance, therefore we use the ratio of Resources to Performance to evaluate the system's effectiveness.

  The other metric is the inequity index. Since the two metrics may not have the same magnitude, it is not appropriate to add or multiply them.

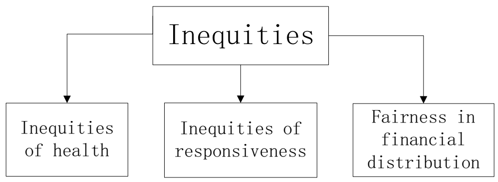

Figure 4. Inequities.

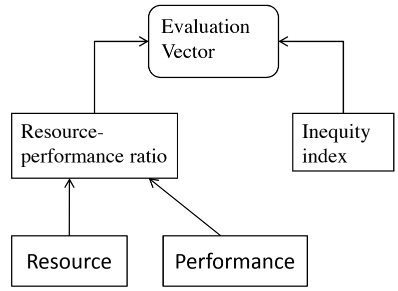

Figure 5. Evaluation.

Hence, we form an evaluation vector (EV) consisting of the two metrics:

$$\text{EV} = \left(\frac{\text{resources}}{\text{performance}}, \ \text{inequities} \right).$$

This is our final composite measure to evaluate the effectiveness of a healthcare system. When both components of the vector are lower, the system is better.

Determine Weights

We specify the calculation of one metric, Resources; the others can be calculated in the same way. After comparing the effect of two criteria in the same layer to the higher layer, we can construct the conjugated-comparative matrix with Saaty's Rule [Jiang 1993]. For example, a_{12} can indicate the difference of the effect on Resources between Human Resources and Financial Resources. Let M_1 be the conjugated-comparative matrix of Resources, while the elements of M_2 are Financial Resources:

$$M_1 = \begin{bmatrix} 1 & 2 & 3 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{3} & \frac{1}{2} & 1 \end{bmatrix}, \qquad M_2 = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 1 & 1 \\ \frac{1}{2} & 1 & 1 \end{bmatrix}.$$

After calculation of the matrix using the summation method [Jiang 1993], we obtain the weight vectors:

$$w_1 = (.539, \ .297, \ .164), \qquad w_2 = (.41, \ .33, \ .26).$$

So we can form the formulas:

$$\text{Resources} = .539 \times \text{FR} + .297 \times \text{HR} + .164 \times \text{MR},$$
$$\text{FinancialResources} = .41 \times \text{Asp}_1 + .33 \times \text{Asp}_2 + .26 \times \text{Asp}_3,$$

where our notations are defined in **Table 1**.

Table 1.
Symbols used.

Abbreviations	Meaning
HR	Human resources
MR	Material resources
FR	Financial resources
HL	Health level
HSC	Health service coverage
RL	Responsiveness level
DALE	Disability-adjusted life expectancy
HALE	Health-adjusted life expectancy
IMR	Infant mortality rate
IH	Inequities of health
IR	Inequities of responsiveness
I	Inequities metric
R	Responsiveness metric
FFD	Fairness in financial distribution
Asp_i	Seven aspects of responsiveness
HL	Health level
RP	Resources/performance ratio
EV	Evaluation vector
L	Length of the evaluation vector
TH	Total expenditure on health as % of GDP
GHtoPH	Ratio of government expenditure on health care to private expenditure
GHtoG	Government expenditure on health as percentage of total government expenditure

Formulas

Using a similar method, we arrive at equations as follows:

$$\begin{aligned}
\text{Performance} &= .49 \times \text{HL} + .31 \times \text{HCS} + .2 \times \text{RL}, \\
\text{HealthLevel} &= .6 \times \text{DALE} + .4 \times (1 - \text{IMR}), \\
\text{Responsiveness} &= \frac{1}{7} \sum_{i=1}^{7} \text{Asp}_i, \\
\text{Inequities} &= .4 \times \text{IH} + .4 \times \text{IR} + .2 \times \text{FFD}.
\end{aligned}$$

With these formulas and our basic criterion, we easily get the evaluation vector to evaluate the effectiveness of a healthcare system.

Strengths and Weaknesses

The Analytical Hierarchy Process method is a good combination of qualitative and quantitative analysis, and it gives the weights conveniently. But it possesses a certain subjectivity.

Solution of Task II

Modify the List of Metrics and Calculate Each

In Task I, we listed three total metrics and several small metrics. But data for some metrics are unavailable, so we need to modify our list of metrics. In this task, we take the U.S. as an example.

Data Disposal

For the sake of consistency, we need to process the original data, which we denote as V_{original}.

Step 1: Find the maximum and minimum values in the whole table, denoted by $V_{\max}$ and $V_{\min}$. The adjusted value is

$$V_{\text{adjusted}} = \frac{V_{\text{original}} - V_{\min}}{V_{\max} - V_{\min}}.$$

Step 2: If the metric has only one factor, we can simply use V_{adjusted}. If the metric consists of several factors, we should give each one the weight as determined in Task 1.

Neglected Metrics

We neglect the metrics of responsiveness inequities and fairness of financial contribution because we lack data.

To quantify responsiveness, WHO surveyed 35 countries, giving scores in seven aspects; but data for the U.S. are absent [WHO 2007]. Thus, we delete this factor. Without the metric of responsiveness, we should adjust the weights in calculating the metric Performance:

$$\text{Performance} = .613 \times \text{HL} + .387 \times \text{HCS},$$

Selected Metrics

- Resources
  - Human resources (**Table 2**):

    $$\text{HR} = .25(\text{physicians} + \text{nurses} + \text{dentists} + \text{pharmacists}),$$

    where the numbers are measured per thousand of population.
  - Material resources (**Table 3**): We choose hospital beds per 10,000 population to reflect the amount of material resources.

Table 2.
Human resources (per thousand of population).

Year: 2000	Physicians	Nurses	Dentists	Pharmacists
U.S.	2.56	9.37	1.63	0.88
Max, 35 countries	5.91	15.2	1.63	3.14
Min, 35 countries	0.02	0.11	0	0
Normalized U.S. value	.43	.61	1	.28

Table 3.
Material resources (hospital beds per 10,000).

Year: 2003	Beds
U.S.	33
Max, 35 countries	1324
Min, 35 countries	2
Normalized U.S. value	.24

– Financial resources (**Table 4**):
  * TH = Total expenditure on health as % of GDP
  * GHtoPH = Ratio of government expenditure on health care to private expenditure
  * GHtoG = Government expenditure on health as percentage of total government expenditure.

$$\text{FR} = \text{Financial resources} = .33\text{TH} + .41\text{GHtoPH} + .26\text{GHtoG}.$$

Since by the usual calculation the normalization result for GHtoPH turns out to be extremely exceptional, we calculate it instead by

$$V_{\text{adjusted}} = \frac{\ln V_{\text{original}} - \ln V_{\min}}{\ln V_{\max} - \ln V_{\min}}.$$

Table 4.
Financial resources as percentage of GDP.

Year: 2004	TH %	GHtoPH %/%	GHtoG %
U.S.	15.4	44.7/55.3	18.9
Max, 35 countries	16.6	98.8/1.2	33.4
Min, 35 countries	1.6	12.9/87.1	1.4
Normalized U.S. value	.92	.27	.55

- Performance
  - Health level (**Table 5**):
    - Disability-adjusted life expectancy (DALE): In our data, there is no information about DALE. So we use HALE, health-adjusted life expectancy, to substitute for it.

$$\text{HL} = \text{Health level} = .6\text{HALE} + .4(1 - \text{IMR}).$$

    - Infant mortality.

Table 5.
Health level.

	HALE (2002) Male	Female	Ave.	Infant mortality (2005) per 1000 live births
U.S.	67	71		7
Max, 35 countries	72	78		165
Min, 35 countries	27	30		2
Normalized U.S. value	.89	.85	.87	.031

  - Health service coverage (**Table 6**):
    We choose percentage of immunization coverages to evaluate the level of health service coverage, plus TB treatment success:
    - Measles = immunization coverage among one-year-olds with one dose of measles
    - Diphtheria = immunization coverage among one-year-olds with three doses of diphtheria, tetanus toxoid and pertussis (DTP3)
    - HepB3 = immunization coverage among one-year-olds with three doses of Hepatitis B (HepB3)
    - TB = tuberculosis treatment success (%)

$$\text{Coverage} = .25(\text{Measles} + \text{Diphtheria} + \text{HepB} + \text{TB}).$$

Table 6.
Health service coverage (percentages).

	Measles (2005)	Diphtheria (2005)	HepB3 (2005)	TB (2004)
U.S.	93	96	92	61
Max, 35 countries	99	99	99	100
Min, 35 countries	23	8	20	9
Normalized U.S. value	.92	.97	.91	.61

- Inequities

  We choose probability of dying aged < 5 years per 1,000 live births (under-5 mortality rate) by place (rural or urban). To our disappointment, data are not available or not applicable Africa, the Americas, and Europe. Therefore, to analyze the healthcare system in the U.S., we use "infant mortality by race" to indicate inequity.

Table 7.

Health inequity in the U.S.: Under-5 mortality in 2004.

	Under-5 mortality	Normalized (relative to Black/AA)
White	5.7	.12
Black or African-American	13.2	1
American Indian or Alaska Native	8.4	.44
Asian or Pacific Islander	4.7	.09
Hispanic or Latino	6.5	.67

Table 7 shows high variability, indicating disparity among races and consequent severe health inequity.

Comparison of Healthcare Systems

We construct two models to compare the effectiveness of healthcare systems.

Model 1

Let EV_i be the evaluation vector of system i: $\text{EV}_i = (R_i, I_i)$, where R_i is ratio of Resources to Performance and I_i is the inequity index.

Design of the Model

We construct the comparison function

$$f(\text{EV}_1, \text{EV}_2) = \frac{R_2 - R_1}{\max(R_1, R_2)} + \frac{I_2 - I_1}{\max(I_1, I_2)}.$$

The first term is the relative disparity of resources-performance ratio between two systems. The second term is the relative disparity of inequity index between two systems.

If $f(\text{EV}_1, \text{EV}_2) > 0$, then system 1 is better than system 2.

Model Expansion

In our function, the two metrics—resources/performance ratio and inequity index—have equal weight. They could be weighted otherwise.

Model 2

Basic Assumption and Symbol Definition

As before, EV is the evaluation vector with components R (ratio of Resources to Performance) and I (index of inequity). The length of the vector, $L = \sqrt{R^2 + I^2}$, measures the effectiveness of the healthcare system.

Basic Model

All the points on the same circle have the same length (**Figure 6**); in other words, the systems have the same effectiveness. Consequently, a system could adjust internal resources distribution; it could sacrifice the resources/performance ratio to improve the inequity index, or vice versa.

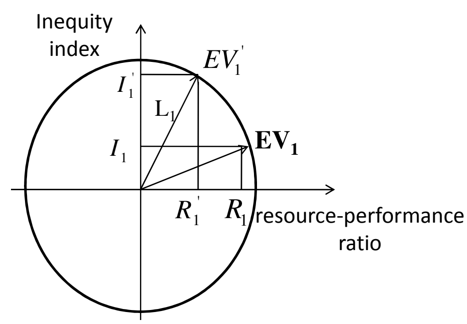

Figure 6. Two healthcare systems of equal effectiveness.

To compare systems, we draw concentric circles according to the evaluation vectors. A system with a smaller circle is better.

Strengths and Weaknesses

Model 1

- The calculation in Model 1 is simple and clear. The model can be easily understood.
- Model 1 could be used to compare any two healthcare systems.
- The weights of resources/performance ratio and inequity index can be adjusted flexibly.

Model 2

- Compared to Model 1, Model 2 is more visual and intuitive.
- Further development of Model 2 can deal with two indexes not of the same order of magnitude. [EDITOR'S NOTE: We omit this elaboration.]
- In Model 2, the weights of resources/performance ratio and inequity index are equal.

Solution of Task III

The Incomplete-Induction Model

In Task 2, we modified the list of metrics. However, some metrics are not so important. We now use the Incomplete-Induction Model to select them most important metrics.

We select metrics that are applicable to most countries' systems. According to the WHO [2001], the metric Inequities is indispensable in evaluating the effectiveness of health system. So we need to choose other metrics only from among the 14 in the two major factors Resources and Performance.

Design of the Model

Step 1: Choose N countries to analyze.

Step 2: For each country i, obtain the resources/performance ratio RP_i^0 (the first component of the evaluation vector) by the method of Task II.

Step 3: Delete the jth metric and calculate RP_i^j using the other 13 metrics.

Step 4: Let $P_j = \sum_{i=1}^{N} \left(\mathrm{RP}_i^j - \mathrm{RP}_i^0\right)^2$.

Step 5: Choose the metrics associated with the two (or more) largest P_js.

Step 6: Some metrics belong to Resources while others belong to Performance. So we need to adjust the metrics if the metrics we have selected are all from Resources or all from Performance.

Result

We choose for our analysis 10 countries, from different regions of different continents, from different levels of development, and with different healthcare systems. In other words, they are representative in the whole world: Argentina, Egypt, Finland, Ghana, Honduras, Japan, Syria, Thailand, and the U.S.

The three metrics with the highest values of P_j are all submetrics of Resources. Consequently, we go with the first two only and substitute for the third the fourth-ranking metric, which is from Performance. Including the metric for Inequity that we regard as mandatory, the set of four metrics is:

- $M_1 =$ total expenditure on health as percentage of GDP,
- $M_2 =$ ratio of public to private expenditure on health,
- $M_3 =$ HALE, and
- $M_4 =$ Inequities.

Application

The resources/performance ratio R can be expressed in terms of the four selected metrics as

$$R = \frac{.446M_1 + .554M_2}{M_3}.$$

where the weights calculated in Task 2 are adjusted through the following method:

$$\frac{.33}{.33 + .41}, \qquad \frac{.41}{.33 + .41}.$$

In Task IV, we discuss how to calculate the metric for inequities.

Measure the Historical Change

We use the four selected metrics to evaluate a system's historical change; we take the U.S. as our example.

The Change of M_1 and M_2

We show the variation trend of M_1 and M_2 in **Figure 7**. Their values increase, which means that the whole nation (especially the government) has attached increasing importance to medical treatment.

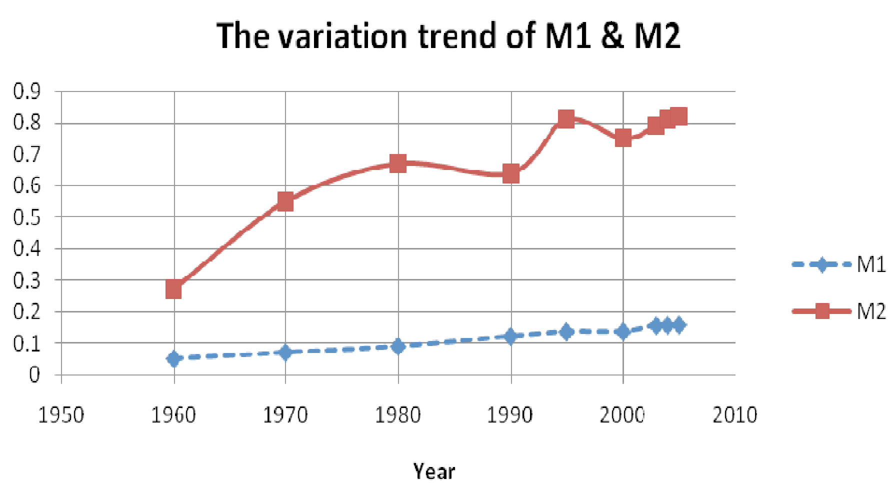

Figure 7. Trends in U.S. total expenditure on health as percentage of GDP (M_1, lower curve) and in the ratio of public to private expenditure on health (M_2, upper curve).

The Change in M_3: HALE

HALE is the most direct and obvious criterion to reveal the health level of the population. Because HALE is a new metric (only since 2000), we can't get enough historical data. Under the circumstances, we use a similar metric, life expectancy, to substitute for HALE. **Figure 8** shows the trend.

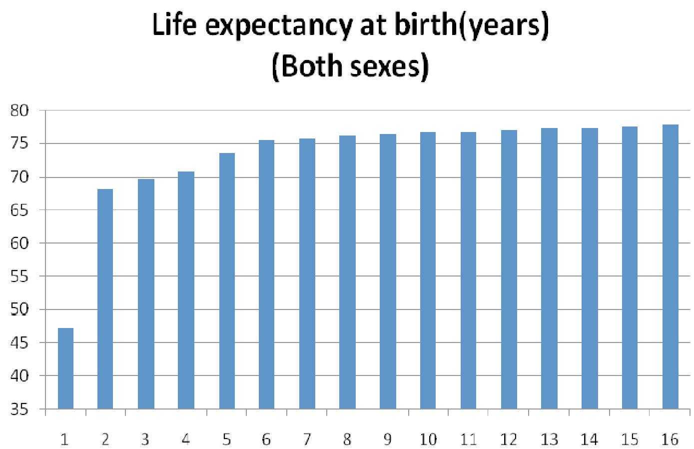

Figure 8. Trend in U.S. life expectancy.

The Change in M_4: Inequities

A good healthcare system aims at not only improvement of the health level but also reduction of health inequity. If the level is reduced or even eliminated, the system is considered to be improved. Recall, we measure inequity in terms of infant deaths per 1,000 live births. **Figure 9** shows improvement in the early 1990s and little change since.

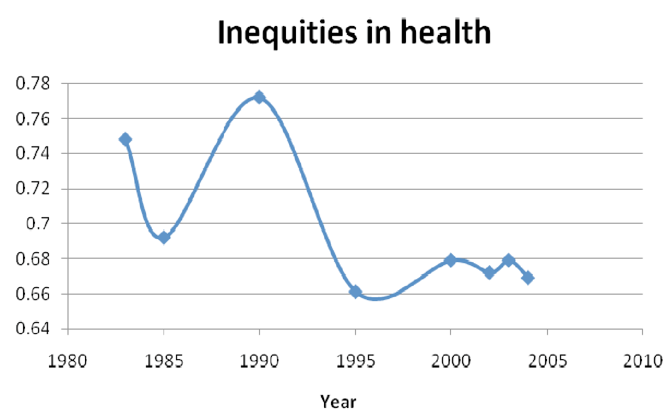

Figure 9. Trend in U.S. healthcare inequity (as measured by infant deaths per thousand live births for different groups).

Solution of Task IV

Brazil's automatic healthcare system is creating enormous value for people there, hence Brazil is considered to have good healthcare.

Calculation of Health Inequities

To measure health inequities in Brazil, we choose three metrics: infant mortality by place (rural and urban), by wealth, and by education level of the mother. In this way, we get three ratios, a_1, a_2, and a_3.

The most equitable situation is if a ratio equals 1; the extent of deviation from 1 shows the unfairness of the system. We use the natural logarithm of original data to normalize the extent of deviation. The bigger the absolute value is, the worse the fairness is:

$$V_{\text{adjusted}} = \frac{|\ln V_{\text{original}}|}{|\ln V_{\max}|}.$$

Adding V_{adjusted} with different weights, we can easily get the index of health inequities of Brazil.

The index of health inequities of India can be calculated in same way.

Comparison

The normalized data for the four metrics for the U.S., Brazil, and India are in **Table 8**.

Table 8.

Comparison of countries.

	% of GDP	Public/Private	HALE	Inequities	EV
U.S.	.92	.27	.82	.67	(.64,.67)
Brazil	.48	.33	.67	.68	(.59,.68)
India	.27	.06	.54	.66	(.24,.66)

The health-adjusted life expectancy in Brazil is shorter than in the U.S., but the U.S. puts more resources into its system in terms of percentage of GDP spent on healthcare. The inequity index in Brazil is a little higher than in the U.S, which means that the distribution of healthcare is more balanced in the U.S.

Using isolated metrics to compare, it's hard to say which system is better. Therefore, we compare using the evaluation vector.

- Compare by Model 1: $f\,(\text{EV}_{\text{U.S.}}, \text{EV}_{\text{Brazil}}) = -0.05 < 0$, so by our comparison principle, the system in Brazil is better.
- Compare by Model 2: $L_{\text{U.S.}} = .92$, $L_{\text{Brazil}} = .91$. Smaller is better, hence the system in Brazil is better.

Solution of Task V

Compared to other countries, India ranks very low in percentage of GDP spent on healthcare, while the U.S. ranks high; moreover, the Indian government stakes little of residents' expenditure on healthcare.

The lack of resources leads to low output. The health-adjusted life expectancy in India is shorter than in the U.S. However, we must take into account that India has much smaller medical resources. The ratio of resources to performance in India is much lower than in the U.S., in other words, India's system is better than the U.S.'s.

In terms of inequities, the two countries are almost at the same level.

Even with increasing resources, the effectiveness of a system won't improve without limit. When the amount of resources is lower than the critical point, the effectiveness of the system will increase sharply as of resources grow. But when the resources are above the critical point, as they increase, the effectiveness of the system grows much more slowly.

Figure 10 shows India at point A and the U.S. at point B. Therefore, India's system has broad prospects for development. To improve the effectiveness of system, India should put more resources into the system, such as increasing the percentage of GDP spent on healthcare, building more hospitals, and adding healthcare workers.

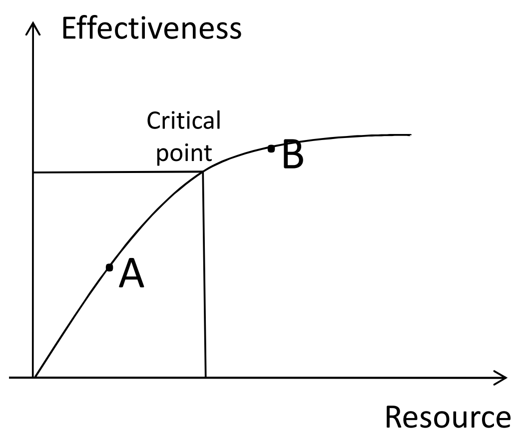

Figure 10. Relative status of the healthcare systems of the U.S. (A) and India (B).

For the U.S., more resources can't bring higher effectiveness. The way to improve the system is to make some change in policy. We discuss detailed measures next.

Solution of Task VI

We choose the U.S. healthcare system to do further study.

Introduction

For the U.S., both criteria R and I are at a high level. But high input doesn't return corresponding high output. "The reasons for the especially high cost of healthcare in the U.S. can be attributed to a number of factors, ranging from the rising costs of medical technology and prescription drugs to the high administrative costs resulting from the complex multiple payer system in the U.S." [Bureau of Labor Education 2001]. So we need to restructure the system based on our four metrics.

Restructuring the System

Modeling: With the four metrics, we can simplify the healthcare system as in **Figure 11**.

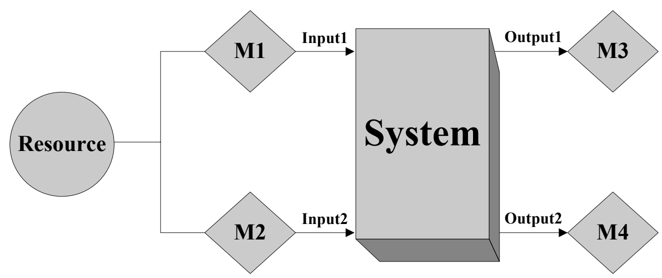

Figure 11. Simplified U.S. healthcare system.

Suppose the initial evaluation vector is

$$\text{EV}_0 = \left(\frac{\text{res}_0}{\text{per}_0}, I_0 \right).$$

The quantity res_0 is determined by M_1 and M_2, while M_3 and M_4 reflect the levels of per_0 and I_0, respectively.

Since M_1 and M_2 are inputs of the system while M_3 and M_4 are outputs, we can describe the system with two functions:

$$M_3 = f(M_1, M_2), \qquad M_4 = g(M_1, M_2).$$

Simplifying the Model

Important considerations are:

- Life expectancy (M_3) is more sensitive to change in total expenditure on health (M_1) than inequities (M_4) is.
- Altering ratio of public expenditures to private (M_2) produces a more sudden response in inequities (M_4) than in life expectancy (M_3).

Thus, the model can be simplified to two single-variable functions:

$$M_3 = f(M_1), \qquad M_4 = g(M_2).$$

Constructing the Functions

M_3**, Life Expectancy**.
The U.S. spends 15.4% of GDP on health, which is the highest percentage in the world. The input and output of its health system have reached saturation. Despite putting more resources into the system, we get little more output, which doesn't match the high input.

For a health system, the growth rate is low when the input (expenditure) is too small or too large but high when the input is appropriate. So we choose the logistic model to describe the function for M_3:

$$M_3 = \frac{ab}{b + (a - b)\exp(-cM_1)}.$$

The value of the function is b when the independent variable is 0, which stands for the HALE when a country spends none of its GDP on health. We use the HALE of year 1900 for the U.S., so we take $b = 47.3$. The value of the function is a when the independent variable goes to infinity, which stands for the saturation of HALE. The highest expectancy life now is about 78, thus we take $a = 80$. We use data from 2004 and get $c = 0.201$. Therefore,

$$M_3 = \frac{80 \times 47.3}{47.3 + (80 - 47.3)\exp(-0.201M_1)}.$$

M_4**, Inequities**.
In our opinion, M_4 will decrease as M_2 (ratio of public to private expenditure) increases. For the sake of convenience, we select an inversely proportional function:

$$M_4 = \frac{k}{M_2}.$$

We use data from 2004 and get $k = 0.548$. Therefore,

$$M_4 = \frac{0.548}{M_2}.$$

Putting Forward Measures

We consider several measures that alter one of the two inputs or both. Accordingly, the two outputs vary.

1. Altering the ratio of government expenditure on health to private expenditure. In the U.S. system, the main use of government expenditure on

health is to improve the health level of low-income people. Altering this can change the level of inequity.

2. Limiting the rise of total expenditure on health as percentage of GDP to make it constant at an acceptable level. Though there is a sharp increase of total expenditure on health as percentage of GDP, the health level doesn't improve much. That is to say, it has reached a saturation point.
3. Limiting the items and the extension of public insurance. In the existing system, public insurance covers a lot of items, some of which may be unnecessary.
4. Increasing the coverage of public insurance.
5. Limiting strictly the use of new medicine, medical equipment, facilities, and medical technology. Research on these has cost too much, and some outcomes are not so important in improving the overall health level.
6. Regulating the cost of medicine.
7. Reducing excessive medical treatment.
8. Promoting positive competition between different hospitals to reduce the patient's cost on medicine and medical treatment.

All these measures can be divided into three groups by their different effect on the inputs:

- Group A (affect only M_1): Measures 2, 3, 5, 6, 7, 8
- Group B (affect only M_2): Measure 1
- Group C (affect both M_1 and M_2): Measure 4

Testing Various Changes

Maybe some measures can improve the healthcare system while others have the opposite effect. Therefore, we have to quantify how each kind of measure affects the system.

In Task 4, we got the evaluation vector for the U.S. In this Task, we take M_1 and M_2 as the inputs of the system and M_3 and M_4 as the outputs. Because we are analyzing only one country without comparing it to another, we can't normalize the original data. If we calculate the vector as same as Task 4, it may lead to abnormal data. So it is necessary for us to modify the calculation method.

$$\mathrm{EV} = (R, I): \qquad R = \frac{M_1}{M_3}, \qquad I = \frac{1}{M_2}.$$

So the initial evaluation vector of the U.S. is:

$$\mathrm{EV}_0 = (R_0, I_0) = (.206, .67).$$

- Measures in Group A can affect only the total expenditure on health as percentage of GDP (M_1). Suppose that its initial value changes by 5%. Calculating M_3 gives:
  - If $+5\%$: $\text{EV}_1 = (.214, .67)$.
  - If -5%: $\text{EV}_1 = (.196, .67)$.

  Hence, decreasing the total expenditure on health as percentage of GDP reasonably can improve the healthcare system of the U.S.

- The measure in Group B affects only the ratio of public expenditure to private (M_2). Suppose that its initial value changes by 5%. Calculating M_4 gives:
  - If $+5\%$: $\text{EV}_1 = (.206, .638)$.
  - If -5%: $\text{EV}_1 = (.206, .705)$.

  Hence, increasing the ratio of public expenditure to private can improve the healthcare system of the U.S.

- The measure in Group C affects both M_1 and M_2. Suppose that the initial values change by 5%. Calculating M_3 and M_4 gives:
  - Case a: If $M_1 + 5\%$ and $M_2 + 5\%$: $\text{EV}_1 = (.214, .638)$.
  - Case b: If $M_1 + 5\%$ and $M_2 - 5\%$: $\text{EV}_1 = (.214, .705)$.
  - Case c: If $M_1 - 5\%$ and $M_2 + 5\%$: $\text{EV}_1 = (.196, .638)$.
  - Case d: If $M_1 - 5\%$ and $M_2 - 5\%$: $\text{EV}_1 = (.196, .705)$.

  Evidently Case c is the best and Case b is the worst.

  The measure in Group C is coverage of public medical insurance. Increasing it on the one hand increases total expenditure of GDP but on the other hand also increases the ratio of public expenditure to private. So such an increase is similar to Case a.

Strengths and Weaknesses

We have built a model that reveal how the system works based on the four metrics that we created in Task 3. Its parts combine well. Also, it is easy and convenient to test the measures with the model. But there are some weakness when simplifying the model. A single-independent-variable function is not the best to describe a healthcare system.

Suggestion: Major Change

From the results above, we find two major changes that can improve the system:

- Decrease total expenditure on health as percentage of GDP.
- Increase the ratio of government expenditure on health to private expenditure.

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From left to right: Advisor Qing Zhou, team members Luting Kong, Yiyi Chen, Chao Ye, and advisor Zuguo He.

Better Living through Math: An Analysis of Healthcare Systems

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Summary

Compelled by the great disparities among healthcare systems across the globe, we create a mathematical model to predict key areas for improvement in stunted healthcare systems. We first establish a framework for discussing and comparing healthcare systems; using data taken from the World Health Organization, we use this framework to rank the systems of the U.S., Sweden, and Nigeria. Our rankings agree with previous studies.

Using a probabilistic model incorporating economic factors, we investigate the effects of various changes to the U.S. system and develop a strategy to improve its rank. Our results indicate that the U.S. should place more emphasis on the prevention of illness, and it should shift toward a more-centralized system so as to make care more accessible to lower- and middle-class individuals.

Introduction

While the U.S. has historically spent more per capita on healthcare than most other countries, the U.S. has seen little improvement in healthcare, and even the U.S. Congress admits that the system is far from the best [1993]. Although healthcare is a significant voting issue, Americans remain confused as to what the remedy for their healthcare should be [Hitti 2008]. Additionally, recent problems such as medical tourism—traveling

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to foreign countries for healthcare—have reinforced the apparent need for reform [Kher 2006], but uncertainty remains as to what reforms should be implemented.

We provide a guideline for improving U.S. healthcare. We offer a framework for comparing and predicting various aspects of healthcare systems. We define important terms and identify metrics for measuring quality. We use the combined metrics to rank the healthcare systems of the U.S., Nigeria, and Sweden; these rankings agree with previous literature and support the effectiveness of our metrics.

We present a predictive model for a healthcare system that can account for different economic classes. Tests run with this model suggest that putting more emphasis on prevention of illness and shifting toward more-centralized healthcare would greatly benefit the U.S.

Defining Healthcare

What is Healthcare?

Healthcare is the utilization of medical knowledge with the intent of maintaining or restoring an individual's health of body or mind. A healthcare system is a network of facilities and workers with the purpose of administering healthcare to a country's population.

Quality of Healthcare

The quality of a healthcare system should reflect how proficient it is at keeping individuals healthy. However, what is considered healthy can change over time, so we define our terms to accommodate changes in medical opinions over time.

The Organization for Economic Co-operation and Development (OECD), a large organization concerned with improving international living standards, defines quality of a national healthcare system as:

> The degree to which health services for individuals and populations increase the likelihood of desired health outcomes and are consistent with current professional knowledge. [2004]

A health outcome is a measurable statistic associated with some feature of the overall health of a nation. We take desirable health outcomes to be universal, and we classify a health outcome as desirable or undesirable depending on the current consensus of the medical community. For example, an increase in a population's average lifespan should always be desired over a decrease, and fewer smokers in a population should always be desired over more smokers [Peto and Lopez 2000].

Metrics for Assessing Quality

We define a *metric* for the quality of a country's healthcare system as a measurement of something that is capable of impacting a health outcome. A *desirable* metric is associated with a desirable health outcome (e.g., average access to medical care, frequency of contraceptive use, frequency of immunizations), and vice versa for an *undesirable* metric (e.g. occurrence of diseases, waiting times for doctors, unaffordable costs).

Due to the large differences in how healthcare is provided throughout the world, some metrics—especially those impacted by culture or geographic conditions—might be inappropriate for comparisons between nations. That is, for a metric to be an effective measure of quality, it should measure something that is impacted directly by health systems and it should be influenced by as few outside factors as possible.

Quality Criteria

The OECD has identified three primary components of success of any healthcare system:

- promotion of good health,
- prevention of illness, and
- treatment and diagnosis of illness [Kelley and Hurst 2006].

Additionally, the OECD has compiled a list of metrics that best measure the quality of each of these components [2004]. We use a slightly modified version of the OECD's description for a healthcare system; we consider a system to consist of the following components:

Prevention. Since promotion of good health and prevention of illness primarily apply only to healthy populations, we treat these two components as one single component, measured by metrics suggested by the OECD for their prevention and promotion components [2004].

Accessibility. People are kept away from treatment or diagnosis by the lack of proximity of healthcare facilities, unavailability of staff, and the price of care [Feldstein 2006]. A healthcare system cannot be effective if it cannot be reached by its population. Metrics for this parameter should measure the system's ability to accommodate people's needs in these respects.

Treatment. This component is unchanged from the OECD definition; the quality of this component should be measured by metrics suggested by the OECD for their treatment and diagnosis component [2004].

Which Metrics to Use

Two common metrics for healthcare quality are life expectancy and infant mortality rate, but both are influenced by factors beyond the control of a reasonable healthcare system [O'Neill and O'Neill 2007]. Life expectancy can be considered more a measure of quality of *life* than quality of *healthcare*; it does not distinguish between treatable causes of death (e.g., disease) and other causes (e.g., war). Similarly, infant mortality rates are strongly influenced by cultural, social, and educational factors. Because of the outside forces, comparisons made with only these metrics are not reliable [O'Neill and O'Neill 2007].

We follow guidelines of the OECD, which has concluded that an effective metric is best characterized by three things:

> First, it [must] capture an important performance aspect [of the healthcare system]. Second, it [must] be scientifically sound. And third, it [must be] potentially feasible. [2004]

Data for Metrics

The World Health Organization (WHO) offers an abundance of statistics relating to healthcare, which are widely believed to be accurate and unbiased. We rely on the WHO as the primary source for health outcomes associated with our metrics.

Our Metrics

We choose metrics based on the recommendations of OECD [2004] and the availability of data in the WHO database [2008]. We group them by component of health, as set out earlier.

Prevention

Obesity. This metric reflects the emphasis that a healthcare system places on healthy dietary habits as well as the public's desire to adopt those habits. Data for this metric are readily available from the WHO as "Adults aged $>$ 15 years who are obese."

Prevalence of contraceptives. Contraceptives prevent both unwanted pregnancies and the spread of sexually-transmitted diseases. The majority of abortions are performed due to unwanted pregnancies; abortions have substantial long-term consequences in women, both psychologically and medically OECD [2004]. This metric responds to measures taken by a healthcare system to reduce risks of unprotected sex. Data are available from WHO as "contraceptive prevalence rate."

Smoking. Reducing smoking has traditionally been the responsibility of healthcare systems. This metric is a measure of how susceptible the public is to beneficial influence from the healthcare system [OECD 2004]. Data are available from WHO as "prevalence of current tobacco use among adults aged > 15 years."

Immunizations. These metrics quantify how proficient a healthcare system is at preventing and controlling communicable diseases [OECD 2004]. WHO offers data for diphtheria, measles, tetanus, hepatitis B, toxoid, and pertussis immunizations in one-year-olds [WHO 2008]. We take an additional data set for polio immunizations from Earth Trends [n.d.].

Low birth weight. This metric is an indicator of the prenatal care that at-risk mothers receive. It reflects a healthcare system's ability to identify risk factors in patients as well as its capacity for preventing those factors from causing serious harm [OECD 2004]. Data are available from WHO as "low birth weight, newborns."

Accessibility

Abundance of medical personnel. This indicates the availability of professionals capable of administering care to the population. The WHO provides several data sets for this metric, including the proportions of physicians, nurses, midwives, dentists, and pharmacists in the population.

Abundance of medical facilities. This metric measures the proximity to healthcare systems. Data for this metric is limited; the WHO provides data only for "medical beds per 100,000 population."

Affordability for individuals. This metric measures how much money individuals pay for care. Data for this metric are not directly available from WHO but instead we derive them from its "private spending" and "out of pocket spending" statistics.

Treatment

Success of treatments. This metric should reflect a healthcare system's level of care. The OECD suggests using the readmission rates for patients who have suffered congestive heart failure [2004], but these data are not widely available. Hence, we resort to using the "tuberculosis detection rate" and "tuberculosis treatment success" data provided by the WHO as an alternative.

Meta-Metrics

It would be convenient to combine all the metrics in a meaningful way; we propose an algorithm for computing what we call *meta-metrics*. Begin by selecting a healthcare component; for each of the metrics corresponding to this component, do the following:

- Determine the maximum and minimum values of the metric for a large sample of countries; if a large sample not available, then the metric cannot be used reliably.
- Scale each country's datum linearly into the interval $[0, 1]$, where the minimum value is mapped to 0 and the maximum value to 1.
- If the metric is undesirable (e.g., prevalence of obesity), subtract the scaled values from 1 to transform the metric into a desirable metric (e.g., lack of obesity).

Then calculate the average value of all metrics associated with a country and define this number to be the country's meta-metric value for the chosen healthcare component.

A meta-metric represent how well a country performs, on average, relative to the rest of the world for a given healthcare component. A value close to 1 signifies that the component delivers care of the highest quality currently available; a value near 0 signifies that the country delivers some of the poorest quality care. Because of their compactness, meta-metrics are easy to use for comparisons between existing and potential healthcare systems.

Comparing Healthcare Systems

United States

The U.S. is the only developed country that does not employ universal coverage [Torrens 1978]. Instead, healthcare is different for every person, and consists of a loose association of coverage plans provided by private sources, the government and employers. The average middle-class person is usually covered by some sort of insurance and employs a private physician in sole charge of managing the individual's healthcare. Physicians exercise substantial influence on the U.S. system, because of their position in healthcare administration, as well as general tendencies of policy to favor private medical practice. This influence leads to the question of whether or not physicians or the federal government should control healthcare. More pressing issues are also troubling the U.S., as the increasing health budget is yielding little advance in the overall quality of care [Torrens 1978].

To test the effectiveness of the meta-metrics, we compare several countries for which there is a clear ranking of healthcare already established. Based on "financial fairness," the WHO ranked the healthcare systems of Sweden, the U.S., and Nigeria as 12th, 54th, and 180th in the world [2000b].

Meta-metric values, calculated from the metrics and processes described earlier, are given in **Table 1**.

Table 1.
Meta-metrics.

Meta-metric	U.S.	Sweden	Nigeria
Prevention	.68	.79	.54
Accessibility	.61	.80	.23
Treatment	.52	.38	.37

Sweden

Sweden operates a nationalized healthcare system that every citizen contributes to based on a proportion of income. As a result, the OECD asserts that citizens enjoy roughly equal benefits, regardless of economic status [Tengstam 1975]. The system is heavily regulated and is run by the National Board of Health and Welfare, which is responsible for supervising medical care in both the public and private sectors. In addition, this Board is in charge of certifying physicians, nurses, and midwives, and also supervises and reviews the decisions of the County Councils, where most of the responsibility for funding and maintaining healthcare falls [Tengstam 1975]. Anderson [1972] suggests that in many ways the Swedish system is superior to that of the United States because of Sweden's longstanding commitment to, and enforcement of, universal healthcare.

Sweden's average world ranking for healthcare trumps the U.S. in all areas but treatment. However, the treatment meta-metric is calculated using a weak tuberculosis metric.

Nigeria

Nigeria operates a three-tiered health system comprised of a national healthcare system financed by all citizens; government health insurance that is provided for government employees; and firms that contract with private healthcare providers. However, a significant number of Nigerians do not enjoy all the benefits of this system. Like many other African countries, the roots of the Nigerian healthcare system can be traced back to a British colonial era. During this period, the health system was equipped to provide care for only a small portion of the population; the system was never adequately adapted to handle the region's growing population [World Bank 1994]. An additional hindrance in the system is an incredible disparity of wealth between upper- and lower-class citizens [World Bank 1994]. Examples of failures in the health system abound. In one case, a 1985 outbreak of yellow fever devastated a small town (killing more than 1,000 people) despite the fact that a vaccine has been available since 1930 [Vogel 1993].

Compared to the U.S. and Sweden, Nigeria's meta-metrics place it at the bottom.

Strengths and Limitations of Meta-Metrics

Our meta-metrics demonstrate the following advantages:

Flexibility. Additional metrics can be easily incorporated into the meta-metrics.

Relevance. Meta-metrics convey the average performance of a country's healthcare system relative to the rest of the world.

Accuracy. The WHO and our meta-metrics both rank the U.S., Sweden, and Nigeria in the same relative order.

These meta-metrics also demonstrate the following disadvantages:

Data is not concurrent. Data sets reported by the WHO can often be several years older than other data sets.

Demanding. Data are required from a large number of countries in order to determine the worldwide maximum and minimum values for metrics.

Simplicity. It may be wiser to weight the metrics in the calculation of meta-metrics instead of taking just their mean.

A Model for a Healthcare System

Assumptions

We assume that for a given nation:

Wealth is not distributed equally. This is especially true for the U.S. [Wolff 2004], which is the focus of most of our attention.

WHO data for that nation is recent and reliable. This assumption is not entirely valid, since some statistics from the WHO that we use date back to 2000. However, this should be less of a problem as data become more widely and frequently reported.

The healthcare system operates in a consistent way. This is not at all true, but for the sake of simplicity we must assume that the system is predictable.

Meta-metrics accurately reflect the performance of the health system. Our results for the U.S., Sweden, and Nigeria support this assumption for all but the treatment meta-metric.

Certain meta-metrics scale with income. Measures taken by a healthcare system to prevent illness affect all people equally [Torrens 1978]. To account for economic factors, we assume that accessibility and treatment scale with wealth. That is, an individual with more money has an easier time finding care and paying for treatment. This is a gross oversimplification, but it allows the model to convey more information.

Definition of the Model

Let A, T, and P be the country's accessibility, treatment, and prevention meta-metrics. We treat them as probabilities of certain events occurring within the healthcare system:

P**:** the probability that an individual will be in good health;

A**:** the probability that an individual will have access to affordable health-care, should they need it; and

T**:** the probability that a sick individual will be correctly diagnosed and properly treated.

We model a healthcare system as the stochastic process pictured in **Figure 1**, and we repeatedly apply this process to track the flow of healthy individuals through the system.

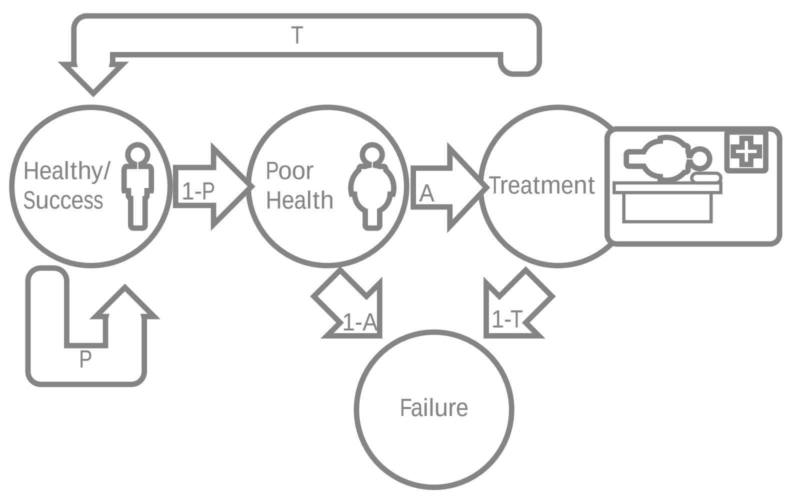

Figure 1. Model of the healthcare process, with four states and probabilities of transitions among them.

If at some time n we have a population of H_n healthy individuals, then we expect $H_n(1-P)$ of those people to fall into poor health in the next time interval. Of those who fall ill, a proportion AT of them will find access to treatment and become healthy. Hence, we predict the number of healthy individuals after $n+1$ units of time to be

$$H_{n+1} = H_n - H_n(1-P) + H_n(1-P)AT.$$

For an initial healthy population H_0, this simplifies to

$$H_n = H_0(P + AT - APT)^n. \tag{1}$$

Retention of the Model

To quantify the efficiency of a healthcare system, we consider how many iterations n of the healthcare process are required before H_n falls below some threshold $H_{\min}$. Hence, we substitute $H_n = H_{\min}$ into **(1)** and solve for n to find the *retention* R:

$$R = \frac{\ln H_{\min}}{\ln H_0 + \ln(P + AT - APT)}. \qquad \textbf{(2)}$$

The retention R measures how long the modeled system can operate, starting from a healthy population, before an overwhelming majority of the population is no longer healthy. A larger retention value indicates a more effective system. For all calculations of R, we take H_0 and $H_{\min}$ to be 100 and 1.

Economic Weighting

One of the primary discriminatory factors of healthcare in the U.S. is economic status; we would like to take this into consideration. To do so, we consider three economic classes:

- Group 1: Those who control the lowest quartile of wealth.
- Group 2: Those in the middle quartiles for wealth.
- Group 3: Those in the upper quartile for wealth,

We adjust the parameters A and T based on the wealth of a group.

Since our meta-metrics describe the average performance of the system, our model—without the economic weightings presented in this section—describes the effect of the system on the "average person," a person of median wealth (hence in Group 2). Analogously, we treat the median person in the lower quartile as a representative of Group 1 and the median person in the upper quartile as representative of Group 3.

We adjust the probabilities A and T for Group 1 by a factor of C_*, the ratio of the median wealth of an individual in the lowest quartile to that of the average person.

Since wealth in the U.S. is so unevenly distributed, comparing the median individual in the upper quartile to the average person would be misleading. Instead, we adjust the probabilities A and T for Group 3 by a factor of C^*, which now represents the wealth of the median individual in the upper quartile with respect to the richest person in Group 3. This gives us a weight based on how the wealth is distributed in the upper quartile.

Simply put, these factors give us a sense of the economic disparity between the groups; quality of accessibly and treatment scales with wealth, and C^* and C_* and appropriate scaling factors. We calculate their values in the **Appendix**.

Let A_i denote the accessibility of the healthcare system for an individual in Group i, and let T_i represent the successful treatment of an individual in group i. We weight each of these probabilities as follows:

$$A_i = \begin{cases} AC_*, & \text{if } i = 1; \\ A, & \text{if } i = 2; \\ A + (1 - A)C^*, & \text{if } i = 3. \end{cases}$$

We will assume that treatment scales in the same way, so we have:

$$T_i = \begin{cases} TC_*, & \text{if } i = 1; \\ T, & \text{if } i = 2; \\ T + (1 - T)C^*, & \text{if } i = 3. \end{cases}$$

The rationale for these weightings is that A and T increase as wealth increases.

By considering healthcare with respect to the actual distribution of wealth, we add a great deal of richness to the model. Adjusted meta-metrics are given in **Table 2**.

Table 2.

Adjusted probabilities for economic classes.

	Class		
	Upper	Middle	Lower
Accessibility A	.87	.61	.11
Treatment T	.82	.52	.09

Analysis of U.S. Healthcare

Applying the data from **Table 1** to **(2)** gives retention values shown in **Table 3**. These values show that the model preserves the earlier rankings.

Table 3.

Retention values.

Sweden	29.6
United States	18.9
Nigeria	8.5

Our interests lie in using this model to make predictive judgments about changes to the U.S. system. To identify the areas in which the retention of the U.S. system is most susceptible to change, we vary one meta-metric while holding the others constant. As seen in **Figure 2**, the slope is much larger for prevention; retention responds more to changes in prevention than in other components. Interestingly, although the U.S. and Sweden have different values for their prevention meta-metrics, they respond essentially identically to changes in prevention.

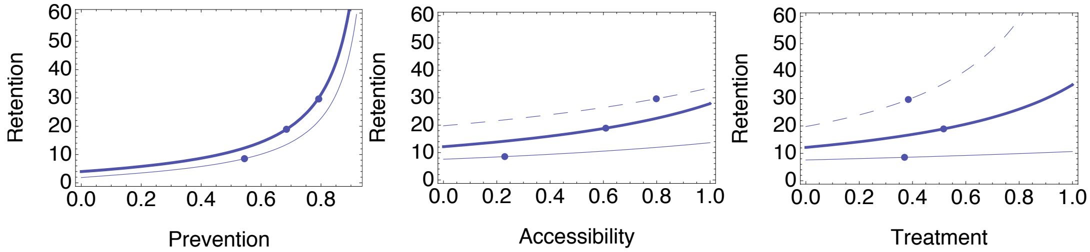

Figure 2. Variations in meta-metrics by nation. In each case, the remaining two meta-metrics are held constant at the values given in **Table 1**. Current values are depicted as dots. Sweden is represented by a dashed line, the U.S. by a bold line, and Nigeria a thin line. Although their prevention meta-metrics differ, the U.S. and Sweden effectively share the same prevention curve.

By considering the impact of economic status on accessibility and treatment, we can gain even more insight. **Figure 3** shows how retention reacts to changes in meta-metrics by economic class: The economic levels in the U.S. react very differently to changes in meta-metrics. This result agrees with out hypothesis that a person's economic status plays a large role in determining the quality of healthcare that the individual receives.

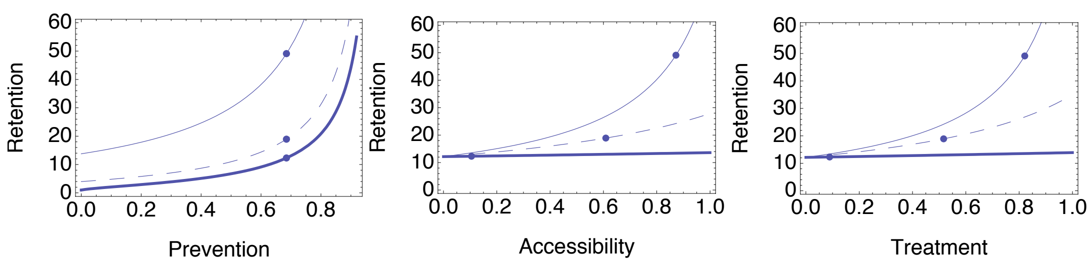

Figure 3. Variations in meta-metrics by U.S. economic class. In each case, the remaining two meta-metrics are held constant at the values given in **Table 2**; for P, we use the value given in for the U.S. in **Table 1**. Adjusted meta-metric values are depicted as dots. The middle class is represented by a dashed line, the lower class by bold line, and the upper class by a thin line.

Strengths

Our model exhibits the following positive characteristics:

Extendability. Our model is a very comprehensive assessment of the interaction of the healthcare system with the population. The advantage of using a stochastic process in creating this interaction is that we can always extend it to be more complex. For example, if we gained access to reliable data for readmission for failed treatments, we could add this pathway into our model and obtain an even more accurate simulation of the healthcare process.

Predictive power. Our model is capable of accurately predicting areas in which national healthcare is lacking relative to other countries, and it can be used to provide insight into the most effective way to change its standing.

Agreement with reality. As discussed later, the results from our model correspond to the current state of U.S. healthcare. Further testing could strengthen this claim.

Economic associations. A large problem with U.S. healthcare is that it varies greatly among individuals, especially by wealth. By incorporating the relationship between availability and treatment into our model, we can more efficiently identify problem areas.

Limitations

Our model also shows the following drawbacks:

Possible failure. It is possible for the model to fail if a country dominates all metrics used in calculating the prevention meta-metric; the model would predict infinite retention. If this occurs, then additional metrics should be considered in the calculation of the prevention meta-metric.

Oversimplification. Our probabilistic model is rather simple, although it produces surprisingly relevant results.

Unconfirmed. Meta-metrics have only been verified to agree with past rankings for a selection of three countries. The accuracy of the model depends directly on the effectiveness of the meta-metrics.

Limited. Our definition for healthcare includes mental health, although our data primarily correspond to physical problems.

Demanding. The model depends on meta-metrics, which in turn require large amounts of worldwide data.

Major Suggestions for the U.S.

Our model predicts that the quickest way to improve the world standing of the U.S. healthcare system is to enhance preventive measures. Lack of spending on the prevention component of the system partially explains the current dilemmas facing the U.S.—namely, the lack of response from increased spending on healthcare [O'Neill and O'Neill 2007] and the growing obesity problem [Wang and Beydoun 2007]. Additionally, it is likely that these inadequate preventive measures are causing more and more Americans to fall into poor health unnecessarily, thereby placing more strain on the system.

We therefore suggest reallocation of funding to place more emphasis on promoting health and preventing illness. **Figure 2** indicates that these changes could quickly increase the quality of U.S. healthcare to be more on a par with Sweden's system.

Additionally, a common criticism of U.S. healthcare is the large inequities in affordability and quality of treatment between the upper, lower, and middle classes [Wolff 2004]. When simple economic factors are combined with our model, **Figure 3** shows that the lower and middle classes experience little to no sensitivity to changes in the system's accessibility or treatment components. At the same time, however, the upper class gains significantly more retention from increases in both of these meta-metrics. Hence, the model suggests that money spent on improving the accessibility component of the system has had a minimal impact on a majority of the population.

Thus, additional reform of U.S. healthcare is needed to make the system more accessible to the lower and middle classes. Sweden has had great success with its highly-regulated universal healthcare system. Therefore, we also suggest that the U.S. grant more control of healthcare to the government so it can enact and enforce stricter regulations on the preventive care provided by private practitioners.

Conclusion

We have researched the motivations and goals of healthcare. Based on quality, relevance and availability, we selected a set of health outcomes that we grouped into metrics, and further organized into logical groups of meta-metrics. Applying these meta-metrics to compare the healthcare systems of the U.S., Nigeria, and Sweden confirmed their validity when considered alongside previous work.

We then used those meta-metrics to construct a stochastic model to generalize healthcare and defined the concept of *retention* to compare different health systems. Furthermore, we incorporated economic factors into our model in order to distinguish between different income classes. By analyzing the influence of each metric on retention, we identified problems in the U.S. healthcare system. In light of these problems, the U.S. system should be restructured to improve promotion of health, and government control should be increased in order to provide more-accessible healthcare for the lower and middle classes.

Future work should seek additional health outcome statistics to increase the accuracy of our metrics, especially for the treatment component of healthcare. Additionally, meta-metric values could be computed for additional countries to further investigate their potential for describing the quality of healthcare.

Appendix

We compute the economic weights C_* and C^* by studying the distribution of wealth within the U.S. Economists often study the distribution of wealth in a country by constructing a *Lorenz curve*, which is the approach we take here.

The Lorenz Curve

The Lorenz curve was described in 1905 by Max O. Lorenz to display the distribution of wealth or assets in a society. A Lorenz curve is obtained by plotting on the x-axis the percentage of people and on the y-axis the corresponding percentage of wealth. Thus, a point at (x, y) on the Lorenz curve indicates the percentage y of total wealth that the bottom $x\%$ of people have. **Figure A1** shows the approximate Lorenz curve for wealth in the U.S. in 2001. The thin line with slope 1 is the line of perfect equality, which corresponds to an equal distribution of wealth among individuals in a society.

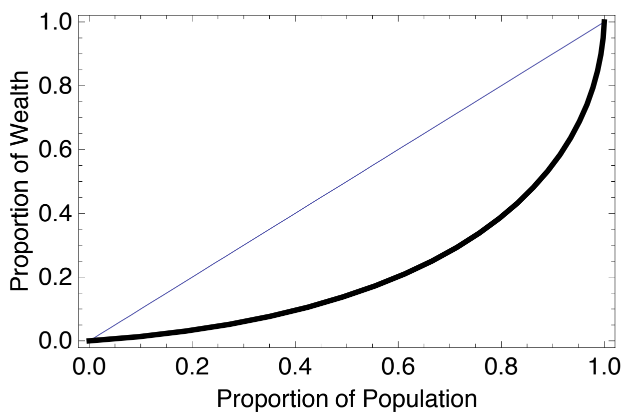

Figure A1. A Lorenz curve for wealth in the U.S. (bold) approximated using data from 2001, along with the line of perfect equality (thin).

A Lorenz curve has properties useful in approximating it:

- It begins at $(0, 0)$ and ends at $(1, 1)$,
- cannot rise above the line of perfect equality,
- is increasing, and
- is convex.

The Lorenz Curve for the U.S.

We approximate the Lorenz curve for the U.S. using data from 2001. We display in **Table A1** the data from Wolff [2004].

Table A1.
Financial wealth distribution by household in the U.S. in 2001, according to Wolff [2004, 30].

	Top 1%	Next 19%	Bottom 80%
% wealth	39.7%	51.5%	8.8%

We also know that 0% percent of the population have 0% of the wealth, and the collective population has all the wealth. This give us the boundary conditions $(0,0)$ and $(1,1)$.

We approximate the Lorenz curve using a Bézier spline fit algorithm because of its ability to generate a smooth curve with relatively few data points. The Bézier fit also guarantees that the curve will be convex, as we would expect a Lorenz curve to be. The disadvantage is that the curve does not pass through all the data points.

Computation of C_* and C^*

We compute the weights C_* and C^* using the Lorenz curve, which we denote by $L(x)$. We define C_* to be the ratio of the cumulative wealth of the median person in the lowest quartile to the cumulative wealth of the average person. Thus, C_* is given by:

$$C_* = \frac{\int_0^{.125} L(x)\,dx}{\int_0^{.50} L(x)\,dx} \approx .17$$

Similarly, define C^* to be the ratio of the cumulative wealth of the median person of the upper quartile to total wealth. So C^* is given by:

$$C^* = \frac{\int_0^{.875} L(x)\,dx}{\int_0^{1} L(x)\,dx} \approx .63$$

The Gini Index of $L(x)$

The *Gini index* is a numerical measure of the distribution of wealth in a country, defined as

$$G = 2\int_0^1 [x - L(x)]\,dx = 1 - 2\int_0^1 L(x)\,dx$$

where $L(x)$ is a Lorenz curve. Thus, the Gini index is 1 minus twice the area below the Lorenz curve. Perfect equality in wealth corresponds to $G = 0$, perfect inequality to $G = 1$. Numerically integration of our function $L(x)$gives $\int_0^1 L(x)\,dx \approx .21$ and hence $G_{\text{USA},2004} \approx .579$.

Limitations of our Approximation

- Although we use data from 2001, the distribution of wealth does not change dramatically from year to year.
- We use only five data points, including the boundary conditions $(0,0)$ and $(1,1)$.
- The Bézier curve passes through the boundary points but not through the data points.

The Gini index for financial wealth of households in the U.S. in 2001 was .888 [Wolff 2004, 30], while our approximation is .579. We used scant data; moreover, the bottom 40% of households combined have negative financial wealth ("net worth minus net equity in owner-occupied housing" [Wolff 2004, 5]). Davies et al. [2008] have different data (**Table A2**) for household wealth, which they take more conventionally to include "non-financial assets [presumably including home equity], financial assets and liabilities" [2008, 2].

Table A2.

Wealth distribution for families in the U.S. in 2001, according to Davies et al. [2008, 4, Table 1].

	Top 1%	Top 5%	Top 10%	Bottom 50%
% wealth	32.7%	57.7%	69.8%	2.8%

Editor's Note: Calculation of the Gini Index from Available Data

The U.S. Census Bureau publishes wealth and income data *by quintiles*. The income data are published separately for families and for households [2005a; 2005b], while the wealth data are published for households only [2008a]. A household includes related family members plus any unrelated people who share the housing unit. The Bureau also publishes Gini indexes for income [2008b; 2007b; 2007a] calculated from the full Lorenz curves, together with other measures of inequality [n.d.].

The Gini index cannot be approximated from quintile data by using Simpson's rule for an integral, since Simpson's rule requires an even number of intervals. Using the trapezoid rule would underestimate the Gini coefficient because of the concavity of the Lorenz curve.

Gerber [2007] gives a simple method suitable to quintile data. For U.S. family income in 2000, the method gives a Gini index of .422, while the index given by the Census Bureau (based on the full Lorenz curve) is .433.

Further information about both the Lorenz curve and Further details about the Lorenz curve and the Gini index are given in a series of UMAP Modules by Schey [1979].

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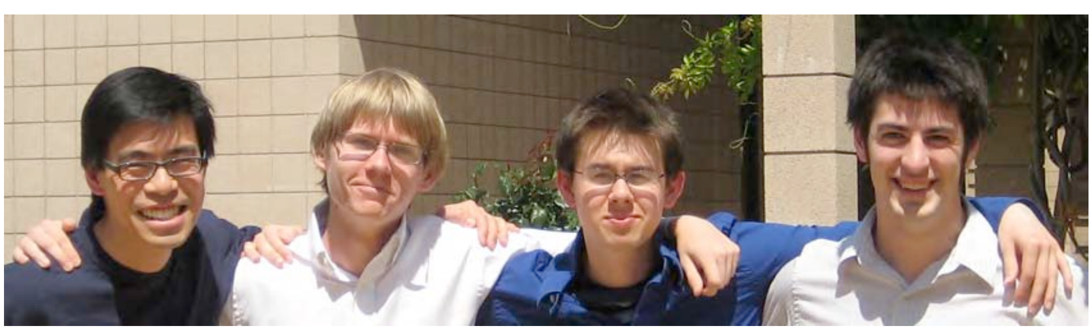

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The Most Expensive is Not the Best

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Abstract

Motivated to evaluate healthcare systems more accurately, we analyze existing evaluation methods. Most methods mainly focus on outcomes and their metrics often ignore internal characteristics of the healthcare systems.

We devise two methods: an improved World Health Organization (WHO) method and a comprehensive evaluation method.

The improved WHO method uses the same metrics as the WHO method, which are determined by the outcomes of the healthcare system. Our improvement is to use a grey comprehensive evaluation and the principle of minimum loss of information to combine the metrics, rather than simply combining them linearly.

In our comprehensive evaluation method, we define five new metrics that concern both outcomes and characteristics of the healthcare system itself, including the effect of the government and the basic situation of a country. Then we use the equal-interval method to get a final score. Compared with other methods, this one does a better job in distinguishing countries and in sensitivity.

After comparing with other four countries that represent the four main modes of healthcare systems, we conclude that the most important reason why the highest cost can't make the U.S. the best is unfairness.

Afterward, we use a neural network algorithm to predict what will happen to the U.S. if some values of the metrics change. We conclude that the U.S. can get the greatest benefit by improving fairness.

We finally consider a policy change, a "medical insurance voucher," as a method to increase insurance coverage and reduce unfairness.

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Introduction

Many countries have recently introduced reforms in the health sector with the explicit aim of improving performance [Mathers et al. 2000; 2001]. There is extensive literature on health-sector reform, and recent debates have emerged on how best to measure performance so that the impact of reforms can be assessed [Goldstein 1996]. Measurement of performance requires an explicit framework defining the goals of a healthcare system and a suitable method to make a compelling evaluation.

So our goal is pretty clear:

- Devise metrics to evaluate the effectiveness of a healthcare system.
- Devise a method to combine the metrics.
- Compare several representative countries.
- Restructure the healthcare system of the U.S. and build predictive models to test the changes.

Our approach is:

- Analyze factors that can affect the performance of a healthcare system.
- Search the literature on existing evaluation methods and find their shortcomings.
- Develop a comprehensive evaluation method that asks only for existing data or data easy to measure and collect.
- Collect experimental data that can be used in our method.
- Compare current methods and determine their characteristics.
- Do a sensitivity analysis of variations of our models.
- Compare healthcare systems of several representative countries.
- Restructure the healthcare system of the U.S. and build a model based on neural networks to test changes.
- Do further discussion based on our work.

Four Representative Healthcare Systems

The healthcare system, as an important part of the social security system, is essential to promote the stability of society, and it reflects social justice. Due to the different histories, cultures, and status of human rights protection, healthcare systems vary from country to country.

There are four representative healthcare systems [Ding 2005]:

- **National insurance.** The main countries using this system are the UK, Eastern Europe, and Russia. The government dominates, healthcare is free, with full medical treatment and complete coverage of the population. But it doesn't have high efficiency or make use of the market, and it is a heavy burden to the government.
- **Commercial insurance.** The U.S. is the main country using this system, which makes the market the guideline of the healthcare system. Cost is high, and a large number of people fail to pay.
- **Social insurance.** This system features mandatory coverage and fairness, as in Japan, Germany, and Canada. It has high cost and slow service.
- **Savings insurance.** Singapore is the representative country. The main disadvantage is a low service efficiency. Costs rise rapidly, and it cannot achieve full coverage.

Analysis of the WHO Estimation Method

The WHO's methods focus on the outcomes of a healthcare system without considering any characteristics of the system itself.

Strengths

The metrics that the WHO uses to evaluate a healthcare system aim to measure goal attainment, and they include most of the outcomes that a healthcare system should produce.

Weaknesses

- The weights placed on each dimension are somewhat arbitrary.
- The approach heavily penalizes countries with epidemic disease unrelated to the healthcare system.
- This approach does not look at how the system is organized and managed.
- The WHO 2000 rankings do not look at access, utilization, quality, or cost-effectiveness.

In addition, according to Almeida et al. [2001, 1693]:

- "The measure of health inequalities does not reflect concerns about equity."
- "Important methodological limitations and controversies are not acknowledged."

- "The multicomponent indices are problematic conceptually and methodologically; they are not useful to guide policy, in part because of the opacity of their component measures."
- "Primary health care is declared a failure without examining adequate evidence, apparently based on the authors' ideological position."
- "These methodological issues are not only matters of technical and scientific concern, but are profoundly political and likely to have major social consequences."

Improved WHO Method

In the WHO methods, the weights in the construction of the composite index are used without considering uncertainty in the values of the different components.

We use a *grey comprehensive evaluation model*[1] to improve the WHO method to make the evaluation more credible.

Methodology

Suppose that c_{ik}, for are the raw data of the metrics $k = 1, \ldots, m$ in country i, for $i = 1, \ldots, n$, giving the $n \times m$ matrix $C = (c_{ik})$. We suppose that c^* is the best value in metric k among all countries. We take $C^* = (c_k^*) = (c_1^*, \ldots, c_m^*)$, a best possible situation, as a reference and compare the value of metric k in country i to this ideal via

$$\xi_i(k) = \frac{\min_i |c_k^* - c_{ik}| + \rho \max_i |c_k^* - c_{ik}|}{|c *_k - c_{ik}| + \rho \max_i |c_k^* - c_{ik}|},$$

where $\rho \in (0, 1)$ is a differentiation coefficient that we generally can take to be 0.5. Using $\xi_i(k)$, we get the evaluation matrix $E = \big(\xi_i(k)\big)_{n \times m}$.

Suppose $W = (w_1, \ldots, w_m)$ is a weight-distribution vector for the m metrics, with w_k the weight of metric k and $\sum w_k = 1$. Based on the discussion above, we get the *grey comprehensive evaluation model*

$$R = W \cdot E^T = (r_1, \ldots, r_n),$$

[1]EDITOR'S NOTE: This method, not known under this name in the U.S., was introduced by Deng Julong in *Tutorial of Grey System Theory* [in Chinese] (1982). It uses ideas of T.L. Saaty's analytic hierarchy process and is well well-known in China (googling `"grey system"` gets 64,100 hits, including *The Journal of Grey System*, edited by Deng). For a numerical example, see Sun, Yan and Zong Sun, The grey comprehensive evaluation model for safety of construction sites, *2007 International Conference on Wireless Communications, Networking, and Mobile Computing*, 5240–5243.

where E^T is the transpose of E and $r_i = \sum_{k=1}^{m} w_k \xi_i(k)$ is the relating degree. The vector $R = (r_1, \ldots, r_n)$ contains the final scores of the countries' healthcare systems. The larger r_i, the better the system.

How to Determine the Weights

We we want to determine the weight vector in a credible way. We use the principle of minimum loss [Wang et al. 2000]. Because our metrics u_j evaluate information from different aspects, combining all the metrics in a linear way would lose a lot of information, according to entropy theory in informatics.

We should maximize conservation of information. So we choose the most classical method: We calculate variance to represent information; the larger the variance, the more information.

In the final score $d = w^T u$, we should choose the best weight w to make the variance of d reach the maximum:

$$D(d) = w^T D(u) w,$$

where $D(d)$ is the variance matrix of d. When $w^T w = 1$, $D(d)$ achieves its maximum.

We use the method of Lagrange multipliers. Suppose that

$$\varphi(w, \lambda) = w^T D(u) w - \lambda (w^T w - 1).$$

Then

$$\begin{aligned} \frac{\partial \varphi}{\partial w} &= 2D(u)w - 2\lambda w = 0, \\ \frac{\partial \varphi}{\partial \lambda} &= w^T w - 1 \qquad\quad = 0, \end{aligned}$$

which reduces to

$$\begin{aligned} D(u)w &= \lambda w, \\ w^T w &= 1. \end{aligned}$$

So λ is an eigenvalue of $D(u)$ with eigenvector w. When $w^T w = 1$, to make $D(d) = w^T D(u) w = \lambda w^T w = \lambda$ reach the maximum, we should take λ as the maximum eigenvalue of $D(u)$.

In the real calculation, we do not know $D(u)$, so we use the variance matrix $\hat{D}(u) = (\hat{\sigma}_{lj})$ of the sample $(c_{1j}, \ldots, c_{nj})$ of u_j to represent it, where

$$\hat{\sigma}_{lj} = \frac{1}{n} \sum_{k=1}^{n} (x_{kl} - \bar{x}_l)(x_{kj} - \bar{x}_j), \qquad \bar{x}_j = \frac{1}{n} \sum_{k=1}^{n} x_{kj}.$$

The variance matrix $\hat{D}(u)$ is a nonnegative symmetric real matrix, so all its eigenvalues are real. From the properties of Rayleigh's entropy, we get

$$\lambda_0 = \max_{w \neq 0} \frac{w^T \hat{D}(d) w}{w^T w} = \max_{||w||=1} \frac{w^T \hat{D}(d) w}{w^T w},$$

where λ_0 is the maximum eigenvalue of $\hat{D}(u)$, and the eigenvector w of $\hat{D}(u)$ is the weight vector that we seek.

A Partial Discussion

The improved WHO method does not change the focus on outcomes of the healthcare system. Its improvement is in making the evaluation more credible. This kind of method makes its own sense in that it really can reflect the goals of the healthcare system, but it can't reflect the inside. For example, a country with an epidemic often gets a low score in WHO's evaluation method, but maybe this is not the problem of the healthcare system. So a new method that reflects the inside is needed.

Comprehensive Evaluation Method

We bring up a method to evaluate a healthcare system, mentioned by [Ding 2005], that considers both the outcomes and properties of systems themselves.

Metrics to Evaluate Overall Effectiveness

To make an overall comparison between countries' health care systems more objectively, fairly and quantitatively, the metrics must be made well. The World Bank has specified the goals of a healthcare system [Schieber and Maeda 1997, 2]:

- "Improving a population's health status and promoting social well-being"
- "Ensuring equity and access to care"
- "Ensuring microeconomic and macroeconomic efficiency in the use of resources"
- "Enhancing clinical effectiveness"
- "Improving the quality of care and consumer satisfaction"
- "Assuring the system's long-run financial sustainability"

Pursuant to this definition, we make five metrics for the overall healthcare system:

- **Efficiency,** the proportionality between inputs and outcomes. It can be divided into technical efficiency, economic efficiency, and allocative efficiency. For our purposes, we choose technical efficiency.
- **Fairness,** both in medical treatment and in contributing to the costs.
- **Responsiveness** "refers to the non-health improving dimensions of the interactions of the populace with the health system, and reflects respect of persons and client orientation in the delivery of health services, among other factors" [Tandon et al. 2000, 2–3].
- **The effect of the government.**
- **The basic situation of a country.** This means a composite index of sectors, which include economy, education, scientific research, and population.

The Model to Deal with the Index and Data

Accordingly, we make five new indexes, one for each metric above.

Choose the Operation Model

We use the method of equal intervals to combine the indexes, which is also used in the Human Development Index by the United Nations to compare countries. We also solve the problem of how to determine the weights.

The Equal Interval Method

The Operating Process

- Divide the subindexes into positive indexes and negative indexes.
- Use different algorithms to make the standardization to the two kinds of indexes.
- According to the subindexes, we can get the five main indexes' composite values.
- Calculate the final score of different countries based on the five metrics' values.

Classification of the Indexes

- **Classification.** Positive index: the higher the value, the better the healthcare system; for example, availability of safe drinking water. Negative index: the higher the value, the worse the healthcare system; for example, the proportion of smokers.

- **Standardization.** The indexes have different units, so we should standardize before calculating the final score. After the classification, we can deal with the two kinds of indexes differently.
  - Positive index: $F_{ilj} = \dfrac{R_{ilj} - R_{il\,\min}}{R_{il\,\max} - R_{il\,\min}} \times 100$,
  - Negative index: $F_{ilj} = \dfrac{R_{il\,\max} - R_{ilj}}{R_{il\,\max} - R_{il\,\min}} \times 100$,

  where
  - i is the one of five metrics,
  - l is the subindex of the metric i,
  - j is the one of the countries,
  - $R_{il\,\min}$ is the minimum value of the l subindex of the metric i in the statistical data, and
  - $R_{il\,\max}$ is the maximum value of the l subindex of the metric i in the statistical data, and F_{ilj} is the value of the l subindex of the metric i after standardization.

Determine the weights. We can get the value of every metric using the function

$$F_{ij} = \left(\sum_{i=1}^{n} \frac{(F_{ilj})^{\alpha}}{n} \right)^{1/\alpha},$$

where n is the number of the subindex in metric i and α is a weight of the metric i.

Get the final score of the evaluated country. Based on the discussion above, we get the function

$$S = \left(\sum_{i=1}^{n} \frac{(F_{ilj})^{\alpha}}{k} \right)^{1/\alpha},$$

where k is the number of metrics (in our case, $k = 5$).

Comparisons between Methods

Before the comparison, each component measure was rescaled on a 0 to 100 scale:

- for healthy life expectancy, $H = \dfrac{\text{Health} - 20}{80 - 20} \times 100$;
- for health inequality, $\text{HI} = (1 - \text{HealthInequality}) \times 100$;

- for responsiveness level, $R = \dfrac{\text{Responsiveness}}{10} \times 100;$
- for responsiveness inequality, RI $= 100(1 - \text{ResponsivenessInequality});$
- for fairness in financing, FF $= \text{FairnessofFinancingContribution} \times 100.$

The overall composite was, therefore, a number from 0 to 100.

Dipartite Degree Analysis

As we know, a good metric should distinguish. But the WHO's method can't; for example, its method gives 36 countries the same value in its metric of responsiveness. We evaluate the degree of distinction via

$$\text{DD} = \sqrt{n_1^2 + \cdots + n_i^2},$$

where $N - i$ is the number of countries that can't be distinguished in criterion i. The smaller DD, the better the degree of distinction.

Monte Carlo Simulation

To test the dipartite degree (degree of distinction) of every method, we use Monte Carlo simulation to make a small change to every data value, since the value must contain some error. The process is as below.

First, we use the beta distribution to determine the change in each value. Because the beta distribution is restricted to the interval $[0, 1]$, a linear function of a beta-distributed random variable can be used to scale the sampling interval appropriately.

The beta distribution can be described by the probability density function

$$\text{Beta}(\alpha, \beta)(x) = \begin{cases} \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\, x^{\alpha-1}(1-x)^{\beta-1}, & 0 < x < 1; \\ 0, & \text{else.} \end{cases}$$

It has expected value $E[X] = \alpha/(\alpha + \beta)$.

Suppose that x_{ij}, with $1 \leq i \leq 191$ and $1 \leq j \leq 10$, is the unknown true mean of the random variable X_{ij} representing the jth metric in country i. We let

$$X_{ij} = (x_{ij} - 1) + 2\text{Beta}(2, 2)(X),$$

which takes values in $[x_{ij} - 1, x_{ij} + 1]$ and has expected value

$$E[(x_{ij} - 1) + 2\text{Beta}(2, 2)(X)] = x_{ij} - 1 + 2[2/(2 + 2)] = x_{ij}.$$

We use Monte Carlo simulation to create 1,000 numbers randomly in the interval $[x_{ij} - 1, x_{ij} + 1]$ and calculate a 95% confidence interval for x_{ij}.

Sensitivity Analysis

About the Values of the Metrics

In this part, we change the values but keep the weights to see how can this change affect the evaluation result. Then we can arrive at the most important metric, the one that can affect the final score acutely.

Suppose that G_p and G_q are the final scores of countries p and q. Let U_{qr} be the the value of metric r in country q. Change it to make $G_p = G_q$; then we can get the marginal value U_{qr}^B:

$$U_{qr}^B = U_{qr} + \frac{G_p - G_q}{w_r}.$$

We can do sensitivity analysis to the values of the metrics following the process below:

- If U_{qr}^B is outside of the allowable interval, whatever it changes, it won't change the order of the two countries; so r is a value-insensitive metric.
- When U_{qr} is close to U_{qr}^B, changing the value will change the order of the two countries; so r is a value-sensitive metric.

About the Weights

In this part, we change the weights but keep the values of the metrics to see how doing so affects the evaluation result. Then we can get the most important weight, the one that can affect the final score acutely.

When a weight changes, it affects others, since the weights sum to 1. To make a simple analysis, when a weight changes, let only one another change at the same time, and keep the others fixed.

Suppose that the weights' values before they change are $\bar{w}_j,\ \bar{U}_{ij},\ \bar{G}_j$ and after changing they are $w_j,\ U_{ij},\ G_j$. Suppose that the changing weights are r and s, so that

$$w_r + w_s = \bar{w}_r + \bar{w}_s.$$

The changing interval of w_r and w_s is $[0, \bar{w}_r + \bar{w}_s]$. When they change, maybe the final score of one country will equal that of another. Let the two countries be p and q. Then we can get the marginal weights

$$\begin{aligned} w_r^B &= \frac{\bar{B}_p - \bar{G}_q}{\left(\bar{U}_{pr} - \bar{U}_{qr}\right) - \left(\bar{U}_{ps} - \bar{U}_{qs}\right)}, \\ w_s^B &= (\bar{w}_r + \bar{w}_s) - w_r^B. \end{aligned}$$

When the two countries have the same score, we can get r and w_s as

$$w_r = \bar{w}_r - w_r^B, \qquad w_s = \bar{w}_s - w_s^B.$$

We can do the sensitivity analysis to the weights following the process below. Because the changing interval of w_r and w_s is $[0, \bar{w}_r + \bar{w}_s]$, if w_r and w_s are outside the interval, the change won't affect the final order of the two countries; the metrics r and s are insensitive. If not, this change may affect the final order of the two countries.

- If $w_r > \bar{w}_r$, so that the weight of metric r is bigger than w_r, the final order of the two countries will be changed; then r is a weight-insensitive metric for the country with the lower score.
- If $w_r < \bar{w}_r$, when the weight of metric r is smaller than w_r, the final order of the two countries will be changed too; then s is a weight-insensitive metric for the country with the lower score.

Analysis of American Healthcare System Based on Neural Networks

The Design of the Back Propagation Network

Because of the difficulty of data collection, we choose just satisfaction and seven other indexes as the inputs to a back propagation (BP) neural network:

- health expenditure per capita,
- number of doctors per thousand people,
- number of sickbeds per thousand people,
- anticipated lifespan,
- infant mortality,
- proportion of the healthcare cost in GDP, and
- extent of healthcare coverage.

So, the network should have 7 nerve cells in input layer, and 15 (= 2×7+1) nerve cells in middle layer.

We choose satisfaction to be the target of the network, so there is just one nerve cell in the output layer. According the principles for designing a BP network, the passing function to the middle layer is a sigmoid function. We created the neural network in Matlab.

Application of the BP Network

To check the effect on satisfaction when an index changes, we make one of them rise by 20% once and keep the others unchanged. Doing that, we can get the satisfaction for each year as shown in **Figure 1**.

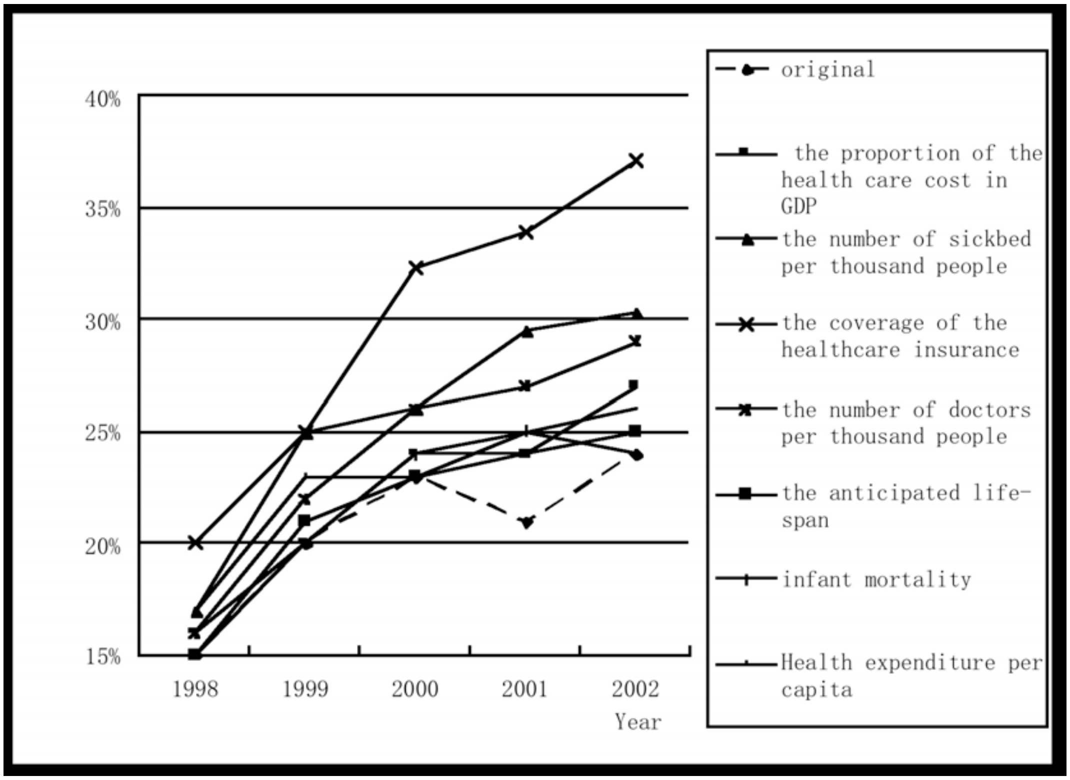

Figure 1. The satisfaction curve after adjustment.

The satisfaction has a rising trend when an index rises. The coverage of healthcare insurance improves the result to the greatest degree. So increasing the coverage of healthcare insurance is a good way to improve the performance of the U.S. healthcare system.

Advice to the Healthcare System of the U.S.

According to the analysis above, the most important problem in the healthcare system of the U.S. is coverage. Though the government has established insurance for the elderly and for children, a lot of people still fail to buy insurance because it is expensive. Universal healthcare coverage will not only lead to fairness in healthcare but also encourage insurers to give better service.

Based on this, we bring up a plan of a “medical insurance voucher” to make the U.S. reach universal healthcare coverage rapidly. We suggest that the government run an insurance institution itself, while at the same time encouraging commercial healthcare insurance institutions. The government should put out the same “medical insurance voucher” to all residents, who can choose a healthcare insurance institution in which to participate.

To fund this program, we would tax smoking and alcohol consumption.

The differences between public and private insurance are in service and cost. The government should provide basic medical care—the lowest level of service. Commercial insurance should offer more service and better conditions, at a slightly higher cost. A resident who participates in commercial insurance should thus pay a little more in addition to using the medical insurance voucher. When healthcare coverage becomes universal, people will pay only a small part of their income to get the healthcare. Advantages of this plan are:

- The plan designs a competitive relationship among insurance institutions, to make them to do their best to reduce cost and improve quality of healthcare—thus improving the effectiveness of the healthcare system.
- In particular, there is a competitive relationship between government (social) insurance and commercial insurance. In some countries where social insurance dominates, needs can't be satisfied and effectiveness is low. Besides, setting social insurance at a minimal level can not only make commercial insurance institutions improve themselves, but adjust the national view.
- Collect the funding for the medical insurance vouchers by taxes, which solves the problem of fairness. Fairness asks the healthcare system not to provide the medical care by income but by need. The tax system has a target of reallocating incomes, and it also can be used to solve the problem of fairness.
- This plan protects the right of choice of residents. It combines competition and human rights, making for a balance between two important problems.

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From left to right: Advisor Ziyang Mao and team members Hongxing Hao, Boliang Sun, and Xiangrong Zeng.

Judges' Commentary: The Outstanding Healthcare Papers

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Introduction

The Interdisciplinary Contest in Modeling (ICM) is a vehicle for students in teams to develop a model to address a problem posed to them, over a four-day weekend. The contest challenges not only students' creativity and modeling prowess, but also their ability to work together in a time-constrained environment. Many of these teams' modeling effort and analysis is truly impressive. In early April, eight judges gathered to read, compare, and contrast the submissions to the contest. Like many of the teams, the judges were an interdisciplinary group with backgrounds in mathematics, statistics, healthcare administration, industrial engineering, and operations research.

The Problem

Teams faced the problem of developing a model to compare many of the world's healthcare systems. This is a timely and relevant issue, particularly in the U.S. with the upcoming presidential election. Although per capita the U.S. spends the most on healthcare, several countries have vastly better health outcomes. In addition, many point to inequities in the U.S. system, resulting

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from a substantial percentage of the population that is either uninsured or underinsured, as evidence that the U.S. system could be improved.

The problem required teams to look at issues faced by countries around the world, including developing a model to help compare the countries' healthcare systems across these issues. The problem tasks fell into four main categories:

- **Metric Identification and Selection.** The teams had to identify metrics for efficiency and effectiveness of healthcare systems and then specify where such data could be obtained. Teams needed to incorporate at least three metrics into their models and justify the importance of the metrics selected.
- **Model Development.** After identifying the metrics, teams had to developing a methodology or model to use these data to compare healthcare systems.
- **Analysis and Comparison of Healthcare Systems.** Teams were to exercise their model to perform at least two comparisons: one to compare the U.S. to a country considered to have a good healthcare system, and a second to compare the U.S. to a country with a poor healthcare system.
- **Recommendations to Restructure a Healthcare System.** After performing the comparisons, teams were to select a country and use their model to suggest changes to restructure its healthcare system. They were either to adapt their existing model or to construct new models to suggest to what degree various changes would improve the system, as measured by the metrics that they had selected.

Overall, the judges were impressed both by the strength of many of the submissions and by the variety of approaches used.

Judging Criteria

To structure the judges' thinking and to ensure consistency across judges, we developed a rubric to evaluate submissions. Its framework encompassed:

- **Executive Summary:** It was important that a team succinctly and clearly explain the highlights of their submission. The executive summary needed to include the metrics selected, modeling approach(es) used, results from comparisons, and recommendations to restructure a healthcare system. Most teams did this. What distinguished the Outstanding papers was the clarity with which the authors connected these topics and conveyed substantial information in the executive summary.
- **Scientific Knowledge and Application:** This task required that teams either have or develop some knowledge about healthcare systems and mechanisms by which effectiveness can be measured. The majority of teams did this relatively well, selecting relevant metrics to use in their analyses. The teams that excelled in this area went beyond traditionally available metrics. For example, some teams combined readily-available metrics into new ones

that would more accurately assess a healthcare system's efficiency or fairness. Others arranged their metrics into groups that would shed light on the analysis. Another way in which teams distinguished themselves in this area was by exploring the tradeoffs between metrics and their limitations. For example, one team noted that although the lifespan of the population was an important metric, various interventions to improve the population's health would take a long time to impact the overall lifespan of the population.

- **Modeling and Assumptions:** The most effective papers made their assumptions explicitly from the scientific foundation that they developed in order to build their models. Models ranged widely in complexity, with factor analysis the most popular approach to synthesizing metrics. It was important that the modeling process was well formulated and robust; but unfortunately, some papers had wonderful models that offered little explanation of how the model functioned or provided little use of the results in the analysis. The ability to use the model to make conclusions and recommendations about healthcare systems distinguished the Outstanding papers regardless of model choice.

- **Analysis/Reflection:** Successful papers discussed how their models addressed issues and tasks of improving the healthcare system in a country. The later requirements of the project were often not addressed, or only superficially. As an example, teams often used models to produce scores to compare countries and conclude that the healthcare system of one was better than another. However, they did not delve into why one country scored higher or to address whether the result was meaningful. In some cases, the final task of restructuring a healthcare system was given very little attention. The best papers used the results of their model to support their recommendations for changes in a healthcare system.

- **Communication:** The ability to communicate effectively really distinguished the best papers from the others. In some cases, the mathematical model was presented with little or no explanation; so, while the work appeared promising, judges could not follow the exposition or determine how the model was used to address the issues. The judges noted several very specific things that made papers stand out, including presenting the work clearly and concisely and effectively connecting the science to the modeling process. Some papers described the healthcare system and issues well but then lost that thread as they began the modeling process. Additionally, some papers were disjointed, possibly because different team members wrote the various sections without ensuring continuity throughout the document.

Discussion of the Outstanding Papers

The Outstanding papers demonstrated true understanding of the difficulties and complexities of healthcare systems, included well-formulated models, and used this work to make thoughtful and interesting suggestions for improving healthcare. While the time constraint of a single weekend meant these papers were not perfect, each team produced work with distinguishing features.

- The Beijing University of Posts and Telecommunications submission (pp. 113–134) is notable for the impressive array of modeling techniques utilized in attacking the problems. There were other papers with a similar level of modeling, but this group not only describes the modeling process clearly but connects the models coherently to the problem at hand. To improve healthcare in the U.S. they propose, among other things, increasing "the ratio of general government expenditure on health to private expenditure" while "decreasing total expenditure on health as a percentage of GDP."
- The paper from the National University of Defense Technology (pp. 155–168) includes perhaps the most comprehensive review of healthcare systems, metrics, and issues among all submissions. The paper is also notable for the sensitivity analysis of its models. Further, this team continues to tie their scientific knowledge throughout the paper, resulting in exceptional comparisons and evaluation of healthcare systems based on their models. They recommend a "medical insurance voucher" to "increase the insurance coverage and reduce the unfairness" in the U.S. healthcare system.
- The Harvey Mudd College submission (pp. 135–154) was among the most clearly and concisely written papers. The team uses "meta-metrics" to map scores on various healthcare metrics into three areas. These then feed into a stochastic model to analyze various changes to a healthcare system. The result is a very strong set of well-supported recommendations for healthcare change in the U.S., such as "emphasis on the prevention of illness," as well as a "shift towards a more centralized healthcare system in order to make care more accessible to lower- and middle-class individuals."

Two other papers, not designated Outstanding and not published in this issue, stood out for the judges.

- The first of these considers healthcare through the eyes and life of "Simon," "an entity who currently does not exist" but "is equally likely to be any person in the world." This paper is not only incredibly clever and creative but demonstrates an outstanding understanding of particularly the economic side of the healthcare debate. The abstract concludes, "Simon says the United States needs healthcare reform now. As we have been told since childhood, it is always good to do what Simon says." Hear, hear.
- The second paper considers the healthcare system through "the lives of John and Jane Doe" and builds a "Virtual Life Model" from first principles. Again,

a very creative and interesting paper! This team notes improvement in the U.S. system but could improve through changes to "prevention, treatment, and access" components of healthcare.

Conclusion

The judges extend their congratulations to all who participated in the contest. The submissions represented not only a variety of approaches that teams used to model the problem, but also a variety of approaches to analyzing the results obtained by the model. Reading your submissions was an enjoyable activity. As judges, we will be excited to see both the types of problems that you approach and the creativity that you use as interdisciplinary modelers after you complete your studies.

Recommendations for Future Participants

- When ideas, assumptions, modeling concepts, and other aspects of the problem are clearly explained, it is easier to separate work that is outstanding from work that is just good. Aim to communicate your ideas clearly and concisely.
- Address all aspects of the problem that are asked. Omitting questions that are asked in the problem statement will not result in a submission that is competitive.
- Simple explanations are usually better than complicated ones. Both clarity and brevity in explanations are preferred to explanations that are long, rambling, and sometimes confusing.
- It is important to cite precise sources for work or words that are not your own. Material taken from other sources must be thoroughly documented and placed in quotation marks.
- The selection of an appropriate modeling approach is critical, but using the model to analyze the problem and present recommendations is often more important than the model itself. Teams should spend time not only in developing the model but in using it to obtain recommendations, analyze different scenarios, and perform sensitivity analysis.
- The recommendations that you make to decision-makers should stem from your model. They should not simply be the result of Internet research or other sources that are independent of your model.
- Team members should work to integrate their final submissions. The judges should not be able to distinguish clear breaks in communication or—worse—identify contradicting information in portions of the paper that were written by different team members.

About the Authors

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Frank Wattenberg is a professor in the Department of Mathematical Sciences at the United States Military Academy (USMA), West Point. He is particularly interested in modeling and simulation and in the use of technology for simulation and for education across the undergraduate curriculum. He is currently leading a team at the USMA that is developing *Modeling in a Real and Complex World* to be published as part of the MAA Online Book Project. He is also working with colleagues at USMA and elsewhere to develop rich immersive environments for modeling and simulation. This project will produce environments with both virtual and hands-on components that students will revisit from middle school through college and from many different subject areas and levels. The architecture will support collaborative modeling and simulation based in part on the ideas of multiplayer games.

Reviews

Tung, K.K. 2007. *Topics in Mathematical Modeling*. Princeton, NJ: Princeton University Press; 336 pp, $45. ISBN 978–0–691–116426.

The world today is awash in textbooks on modeling. These range in level from texts for students with very little mathematical background to texts for graduate students and professionals. Many of these books use modeling as a foil to promote a particular agenda: dynamical systems, or nonlinear differential equations, or perhaps finite mathematics. These books are less interested in teaching and more interested in planting a flag.

The book under review is a refreshing departure from the sorts of polemics just described. Tung's preface shows that he is a dyed-in-the-wool teacher of considerable talent whose only mission is to show the student how to take raw empirical data and turn it into a mathematical paradigm that can be analyzed. His prerequisites are solid but minimal: calculus and a smattering of ordinary differential equations (ODEs). He is wise to provide an appendix with a quick treatment of ODEs for those whose background is deficient. Tung also describes in the preface a clear path for those who wish to avoid the differential equations altogether.

Tung covers some of the usual modeling topics but also many others that are surprising and refreshing. Among the former are

- Fibonacci numbers,
- compound interest,
- radiocarbon dating,
- Kepler's laws,
- nonlinear population models, and
- predator-prey problems.

Among the latter are

- global warming,
- marriage and divorce,
- analysis of the El Niño effect,
- the age of the Earth,

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- the Broughton and Tacoma Narrows bridges,
- climate models,
- HIV modeling, and
- mapping the World Wide Web.

Tung uses a variety of techniques to analyze these different problems. Among these are differential equations, dynamical systems, linearization, phase-plane analysis, and many others. One important feature of the book is that an entire chapter is devoted to each problem and its related ideas. In a calculus class, the student typically sees examples and problems that can be solved in a few lines. Here the student sees the substantive development of mathematical ideas over the course of a prolonged discussion. The book does not contain any proofs per se, but it has discussions that have the *gravitas* of proofs.

The writing in this book is delightful and elegant—almost literary in its beauty and precision. The presentation is thoughtful and readable. The organization is exemplary. As an instance, each chapter begins with a few words telling the reader exactly what mathematics will be needed for the discussion. Every chapter has a useful introduction. There are many interesting references of a philosophical or cultural nature.

The book contains plenty of entertaining graphics, photographs of mathematicians, and other illustrative figures. The sections and their titles are chosen to give the reader a keen sense of the flow of ideas. The layout of the book is open and friendly. This is certainly an inviting text for students.

One of the truly critical components of a successful textbook is the exercise sets. This text contains exercises that are quite thought-provoking. Each is a word problem that could be used for class or group discussion, and many of these problems could be developed into research projects or term papers. One might wonder whether it would have been propitious to include some elementary exercises as well. If I were to teach from this text—and I would certainly enjoy doing so—I would probably find myself hunting around in other texts for routine and drill exercises to give the students. (One may well puzzle over what these elementary exercises might consist of. But *something* must be provided to help bring students up to speed.) That would be too bad, for it is the author's job to provide that sort of material. But I must stress that the exercises that *are* provided are the product of much research and thoughtful editing. They are quite valuable and instructive.

Another small criticism—or at least a comment—is that the book contains few if any displayed examples. One usually expects a textbook to have the format

- Introductory patter, then
- enunciation of idea, followed by

• illustrative example

for each key topic. Tung's book deviates from that paradigm because in fact *each chapter* is a topic. Each *chapter* is an example. It actually does not make a great deal of sense—for what Tung is trying to achieve—to have displayed examples. Such items would be too trite.

But the point of the discussion in the last two paragraphs is worth noting: For many if not most students, this course, and this book, will be a first exposure to serious mathematical discourse. Here, for the first time, the student will see protracted mathematical reasoning directed toward a sophisticated and well-defined goal. This is not the place for cute little problems with three-line solutions. This is instead the venue for rather recondite reasoning. Elementary exercises and elementary examples do not really have a place here. The student in a course like this will need to exert some effort in order to get something of value out of it. But the effort will be well rewarded.

Another important point is that this text illustrates, unlike any text in a previous or more elementary course, the symbiosis of mathematics with other parts of science and technology. Mathematics is *not* a cottage industry that caters primarily to its own whims. Rather, mathematics is the key to understanding much of the world around us. Surely Tung's book illustrates this important point clearly and decisively. That is an incisive message for any student to see and understand, and it takes a good textbook to get the message across.

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Hunt, Earl. 2007. *The Mathematics of Behavior*. New York: Cambridge University Press, 2007; x+346 pp, $80, $34.99 (P). ISBN 978–9–521–85012–4, 978–0–521–61522–8.

For too many mathematics professors, if they have any idea of industry, it is the government research laboratories. That is, they have no idea of the sort of jobs that most people working in industry with mathematics degrees experience. But I am going to give one piece of advice for the student facing industry, and then I will extrapolate from it back to academia.

Suppose that a student with a recent degree in mathematics receives two job offers that seem utterly equivalent in pay, conditions, security, and so on. But there is one significant difference. In Corporation A, the worker will be working with many mathematical scientists in an environment where much of the work is inherently mathematical (and scientific). In Corporation B, the worker will be the house mathematician: the go-to person for mathematical questions.

Now, though the second job might sound like a nice opportunity, the worker is almost certainly better off to take the job with Corporation A. In Corporation A, you have mathematical workers who make work for one another. With any luck, they will support one another and collaborate on papers and generate more mathematical work. But in Corporation B, the worker is very likely to be lonely—the corporate pariah. The mathematical advice that the worker is solicited to provide is likely to be met with incomprehension and suspicion.

How does this apply to academia? Suppose that a student has a degree in mathematics but wants to pursue a Ph.D. in cultural anthropology. My advice is to pursue a degree in physical anthropology instead. Physical anthropology is an area that is much more quantitative and where the mathematically-trained student has a much greater chance of fitting in. After receiving the Ph.D., there is little to keep the physical anthropologist from venturing into cultural anthropology at least occasionally. There is an unmistakable trend towards greater use of mathematics in each of the social sciences. I have even heard of departments in the social sciences recruiting mathematics majors on the grounds that it is easier to convert them to the discipline at hand than trying to convert social science students to mathematical methods. (This sort of thing seems to occur frequently in different areas and is one reason to major in mathematics.)

The book reviewed here is a survey of experimental psychology, a field that has a long tradition of mathematical modeling and in particular of first-rate statistical research. Psychology, as we are told on Day 1 in Psychology 100, is the study of behavior. Earl Hunt has practiced mathematical psychology for 50 years. The book is a fairly comprehensive survey of experimental psychology; it does not touch upon any area of clinical psychology, even when the controversies there may have mathematical content (largely through the use of statistics). I will not enumerate the topics covered in this book. Many of them will be familiar to anyone with a knowledge of applied mathematics. Some of the material will be familiar only to experimental psychologists—for example, the chapter on the physics of perception. That subject does not interest me much, but I found it valuable because of Hunt's historical approach.

The book begins with Eratosthenes' work on estimation of the Earth's circumference. This is because Prof. Hunt first discusses the philosophy of mathematical modeling. His second chapter explores the foundations of probability. I think that he is a little too theoretical here, given the subject of the book; no one is going to learn probability here. He says (p. ix) that the book should be accessible to anyone with a "basic understanding of calculus, and most of the book will not even require that." I would require some calculus for mathematical maturity. I would also expect the reader to have some knowledge of probability and of statistics.

Prof. Hunt, as an experimental psychologist, is quite good on statistical issues. But the reader should have some knowledge of it coming in. To put it succinctly: A freshman with one semester of calculus can learn a lot from this book and should acquire the gist of many topics; a senior with many mathematics courses can learn a lot more and can get a remarkably clear idea of experimental psychology; but knowledge of statistics would be most useful to the reader. There is serious attention to Bayes' Theorem, to covariance, to regression, and to factor analysis (a technique used by experimental psychologists probably more than by any other group). This book will give the mature student the information necessary to make an informed decision about pursuing experimental psychology in a graduate program. One topic in experimental psychology is among the most controversial and dangerous topics in all of academe. I am referring to intelligence testing, which has been the subject of a remarkable amount of political posturing and uninformed commentary, in some cases by mathematicians. Prof. Hunt's discussion of that area is far more detailed and sophisticated than what readers see in the popular press (and by "popular press" I include venues such as *Scientific American*). However, it cannot be considered a comprehensive survey; it is instead a good introduction to a contentious subject at an appropriate level. The wars over intelligence testing are far more intense than those in mathematics and the hard sciences. That controversy has had far more public exposure than most.

A similar war in clinical psychology, over "repressed memory," has involved serious contributions by experimental psychology. Prof. Hunt shows the experimental approach to memory. He barely mentions the controversy and equivocates to some extent; other experimental psychologists have a great deal to say on this topic and there is no hint here of the intensity of the conflict. In fact, a survey of the battles over repressed memory is sufficient to show that conflicts in the social sciences tend to be more intense than those in math and the hard sciences and that social scientists do combat at an entirely different level. A corollary to this is that Ph.D. programs in these areas can be even more hazardous than those in the "hard" sciences. Keeping this in mind, mathematics students should consider the social sciences for graduate study. (This is certainly true of economics. Economics in the U.S. and the U.K. is so mathematical that I think that mathematics majors have an advantage.)

The book reviewed here is a good survey of much of experimental psychology. It is also of interest to anyone pursuing the mathematics of sociology and to a lesser extent political science (for the treatment of Arrow's theorem).

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Nahin, Paul J. *Digital Dice: Computational Solutions to Practical Probability Problems*. Princeton, NJ: Princeton University Press; xi + 262 pp, $27.95. ISBN 978–0–691–12698–2.

The evolution of computer languages is a fascinating but complex topic. But it is probably safe to say that prior to about 1985 most languages were quite useful for numerical computation (although there are obvious exceptions such as COBOL, the dominant business language). However, in the late 1980s, C and then C++ became dominant in computer science departments. The problem with C was twofold:

- it is a cryptic language; and
- it is designed to give the user access to the guts of the machine—a capability largely irrelevant to numerical computation.

C (C++ actually) was replaced as the dominant language by Java. Java was designed for Web development; again, a purpose largely irrelevant to computation.

The result of all of this is that it is easy to encounter students who have passed courses in programming but who somehow do not know anything about simple control structures. Many of them cannot program a spreadsheet. (Spreadsheets can be remarkably efficient for a variety of tasks, especially in discrete mathematics but also in areas such as differential equations.) It appears that mathematicians are responding to this problem by doing their numerical programming in mathematical environments such as Mathematica, Maple, and Matlab. Also, I suspect that mathematics departments are teaching their majors programming themselves rather than sending them to the CS departments.

Nahin's book reviewed here could be a useful for a course in mathematical programming. Paul J. Nahin is is professor emeritus of electrical engineering at the University of New Hampshire. In recent years, he has published a number of books. His *An Imaginary Tale: The Story of i* [1998] is I think the best introduction to complex arithmetic and analysis for the undergraduate in mathematics and the sciences. The sequel, *Dr. Euler's Fabulous Formula* [2006], is something of a tour de force.

In 2000 Nahin published a book of probability problems, *Dueling Idiots and Other Probability Puzzlers* [2000], in which there is sometimes the suggestion of analyzing the problem by simulation. The book reviewed here is explicitly dedicated to that technique. What makes the book useful is that there is little formal probability. Density functions and probability distributions do not come up; so the teacher using this book as a source can concentrate on programming and logic.

The book is divided into four parts: introduction, problems, solutions, and appendices. The solutions usually contain analysis as well as numerical results. Programming is in Matlab, which closely resembles structured Basic and functions effectively as readable pseudocode. The introduction is an informative essay, and the nine appendices are themselves likely to be of interest to the reader.

Another current educational issue is that students in the mathematical sciences often do not seem to understand the power of computation. This is because the curriculum tends to lag behind technology—by decades.

For example, statistical methods, to a remarkable extent, reflect the technology that was available to Ronald Fisher in the 1920s. Many statistical tests can be replaced by remarkably quick computational methods; moreover, these methods often are nonparametric and as such (I would say) esthetically superior to classical methods. The very first problem in *Digital Dice* is essentially a statistical test: Five dishwashers work in a restaurant. In a one-week period, five dishes are broken and four of those are broken by one individual. The individual claims that this is merely bad luck. The problem is to calculate the probability that one individual would break four out of the five dishes under the assumption (null hypothesis) that each dishwasher is equally likely to break a dish. Here, though, I believe that author Nahin is in error. He calculates the probability that the one particular worker would break four or more dishes under the null hypothesis. The correct approach, I am sure, is to calculate the probability that some one employee would break four or more of the five dishes under the null hypothesis. In any case, that very question is a good one for the students to contemplate.

Much of the impetus of *Digital Dice* is the analysis of counterintuitive problems, with the idea that these problems might better motivate the students. There is a lot of truth to that; nothing is more motivating than results that are unexpected and surprising. These problems often motivate both approaches to the solution: analysis to understand why things work the way they do, and simulation to verify both the way things work and the analysis.

In my review of books on investment [Cargal 2006, 87–89], I discussed a problem from Morton Davis and I recapitulated his analysis, which leads to a highly counterintuitive result about a reasonable-seeming investment strategy. When I first encountered the problem in 1989, I was working in industry. To simulate the problem, I would wait until after work and run it on as many as 10 personal computers simultaneously (this was when a 25 MHz Intel-386 processor was considered fast). I didn't doubt the analysis of the problem—it is at the precalculus level—but I had to see it with my own eyes.

The fact is, though, that counterintuitive problems are not necessary. Students at this level usually are not acquainted with the gambler's ruin problem, for example, and this book offers a great set of exercises. Another writer, Julian Havil, has produced a remarkably similar set of books. His *Non-Plussed...* [2007] is devoted to counterintuitive problems; and its sequel—which I believe is of even greater interest to mathematics majors—*Impossible?: Surprising Solutions to Counterintuitive Conundrums* [2008] has just appeared, with a prepublication blurb by Nahin. (Havil's *Gamma* [2003] would roughly correspond to Nahin's *Dr. Euler's Fabulous Formula*.) However, the problems in *Digital Dice* are selected for computation exercises and are not necessarily as counterintuitive as those in the prior book *Dueling Idiots*. The most counterintuitive problem in *Digital Dice* is problem 14: Parrondo's Paradox, probably the most challenging probability paradox I have seen; it has received a fair amount of attention lately. As usual, Nahin both discusses computational simulation and provides an analysis. However, Havil provides probably a better analysis in *Non-Plussed* [2007].

All of the books by Nahin and Havil are worth having, including others not listed here. I particularly recommend *Digital Dice* for the task of teaching undergraduates in mathematics the fundamentals of computation and simulation.

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Shiflet, Angela B., and George B. Shiflet. *Introduction to Computational Science: Modeling and Simulation for the Sciences*. Princeton, NJ: Princeton University Press, 2006; xxiv + 554 pp, $69.50. ISBN 0–691–12565–1.

Applied mathematics is almost synonymous with mathematical modeling—which is why there is an annual issue of this journal devoted to a modeling competition. It might also explain why the editor constantly exhorts me to find books on modeling. Very roughly, we can divide the requisite knowledge for modeling into three areas:

- mathematics,
- applications, and
- computation and programming.

Each area presents challenges to the student, and instructors probably tend to underestimate these challenges—which is why the first course in modeling can be an unpleasant experience for everyone involved. In pedagogical terms, this book is the best thing I have seen in a long time; books this good come along about once every five years.

A great deal of work has gone into this book, and it does many things extremely well. Technically, the book does not require calculus; it introduces the concepts needed, such as rate of change, and in fact does a nice job of motivating the derivative. However, for all practical purposes, calculus 1 should be required. The book is also a fine introduction to differential equations; but again, the student who has had the course has an advantage.

The same can be said about half a dozen other courses, but the course in differential equations is particularly useful, partly for a strange and perhaps controversial reason. Logically, there is a lot to be said for starting with first-order differential equations and Euler's method (and direction fields). However, the *logical* sequence in which students should acquire material and a *pedagogical* sequence—one that actually works in the classroom—are often at odds. This is one of the main challenges of teaching: how students learn and how they should learn are two different things. Differential equations is a great plus before using this text because the student is in a position to appreciate Euler's method (not to mention Runge-Kutta, which is also covered in the text).

Nonetheless a course in differential equations is not necessary at all. Let us assume that the students are sophomores with a semester of calculus. For these students, part of the challenge of modeling is that they must learn the salient features of the modeling context at hand, which may be mechanics, probability, biology, or climatology, etc. As a rule, students feel that they do not get enough information from the instructor about that background. Of course, instructors disagree; but I sympathize with the students. In this regard, the book shines. It has excellent tutorials throughout that are clear, informative, and well-presented; a high school student could learn a lot from the book. Moreover, the book covers such a variety of subjects that any instructor will find pet topics. Since one co-author is a biologist, there is a range of biology projects; but there is also a surprising range of physics and engineering topics, as well as very good treatments of purely numerical issues.

Introduction to Computational Science is divided into 13 sections. However, each section is divided into from 2 to 10 modules. Each module has an introduction and may have downloads, lessons, exercises, projects, references, and other items. The references are fairly thorough; nothing in this book is half-hearted. There is a variety of computational tools rather than concentration on the use of any single tool or language.

I think this text is a masterpiece. I know of nothing comparable. I give it five stars.

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